Predicting Large Scale Wind Farm Power Generation: Effects of Turbulence

(and some illustrations of applications from the field of fluid dynamics and turbulence)

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Simulations: CISL, XSEDE, MARCC, ARCH

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Wind energy and turbulence

does turbulence matter?

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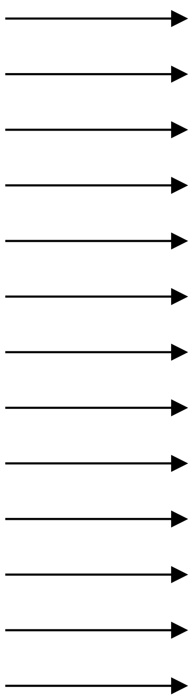
Eiffel tower and turbulence:

Weidman P. "*Modified shape of the Eiffel Tower determined for an ABL wind profile*". Phys Fluids. 2009; **21**(6):067102.

Eiffel assumed plug flow

$$U(z) \sim z^0$$



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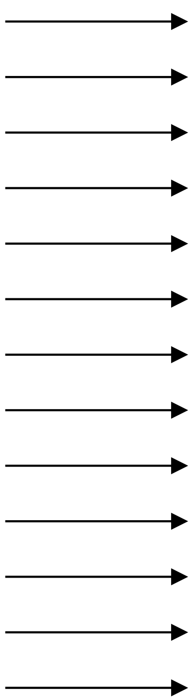
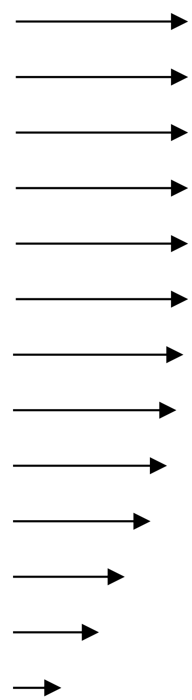
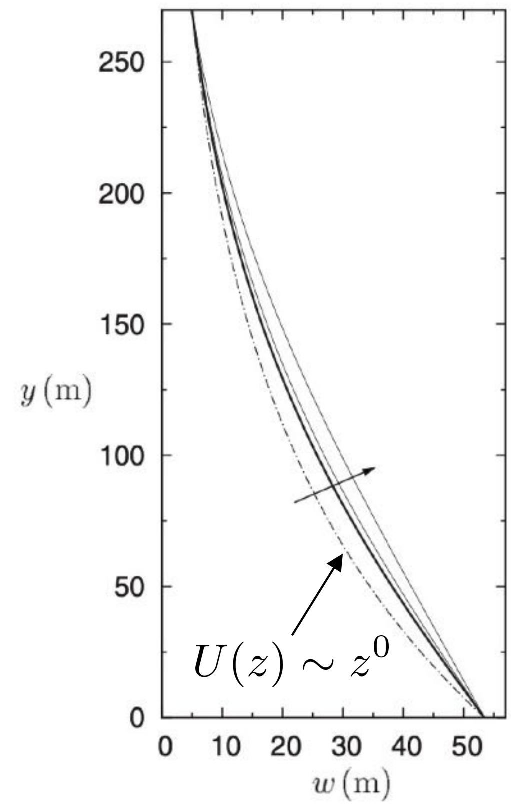


If using power-law profile, tower shape changes

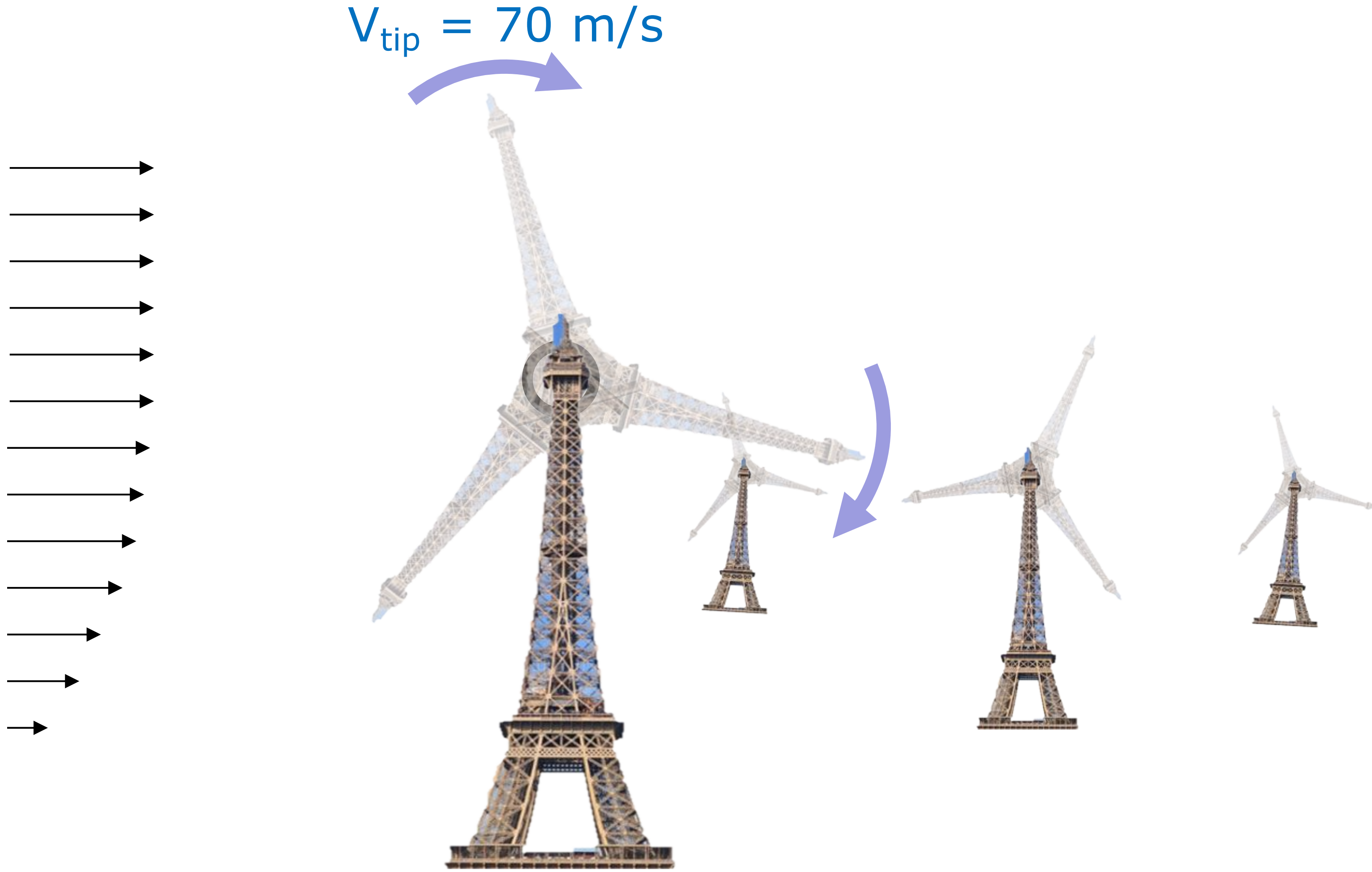
$U(z) \sim z^{1/n}$





Velocity & turbulence profile matters!

Now imagine an **array of dynamic Eiffel towers**



Velocity - turbulence profiles - structure matter !

Power-density

Solar:

Much of US: 6000 kWh/m^2/day = 250 W/m^2

with η = 15%: 40 W/m^2

https://eurasianetwork.eu/2019/01/04/a-109-mw-solar-farm-to-be-built-in-belarus-in-2019/

Power-density of wind farms:

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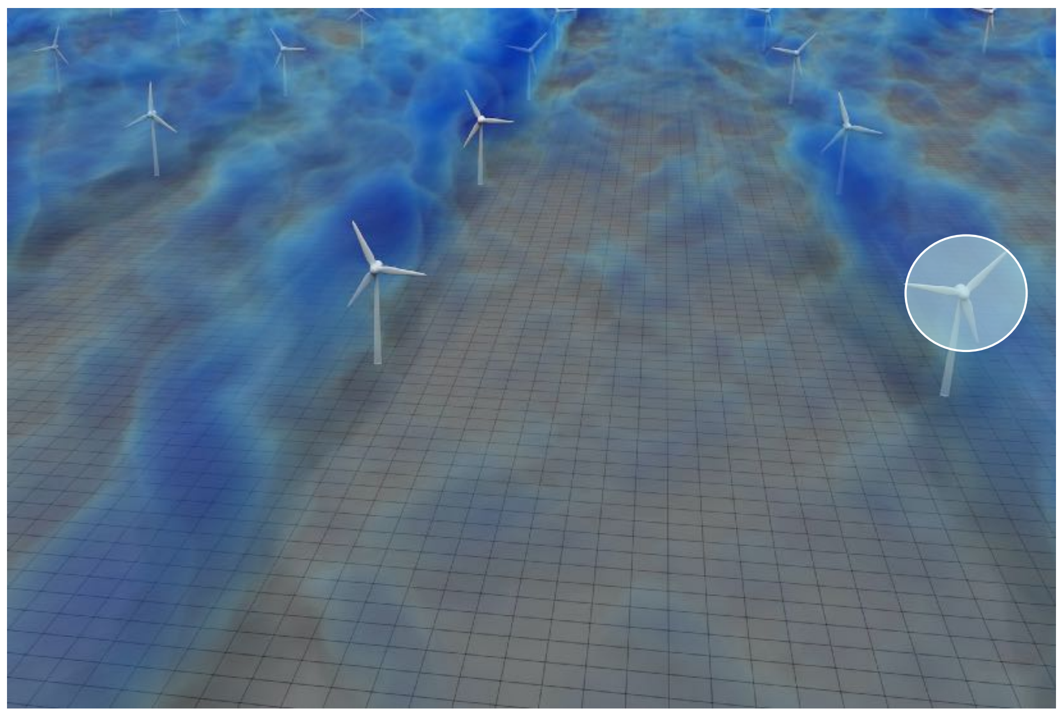

$$\frac{P}{A} = \frac{0.3 \times 5MW}{\pi 60^2}$$

$$= 130 \frac{W}{m^2}$$

Power-density of wind farms:

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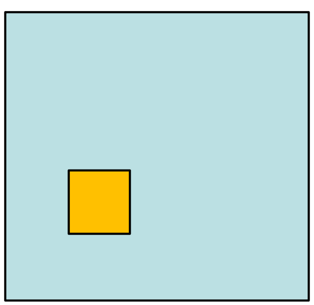
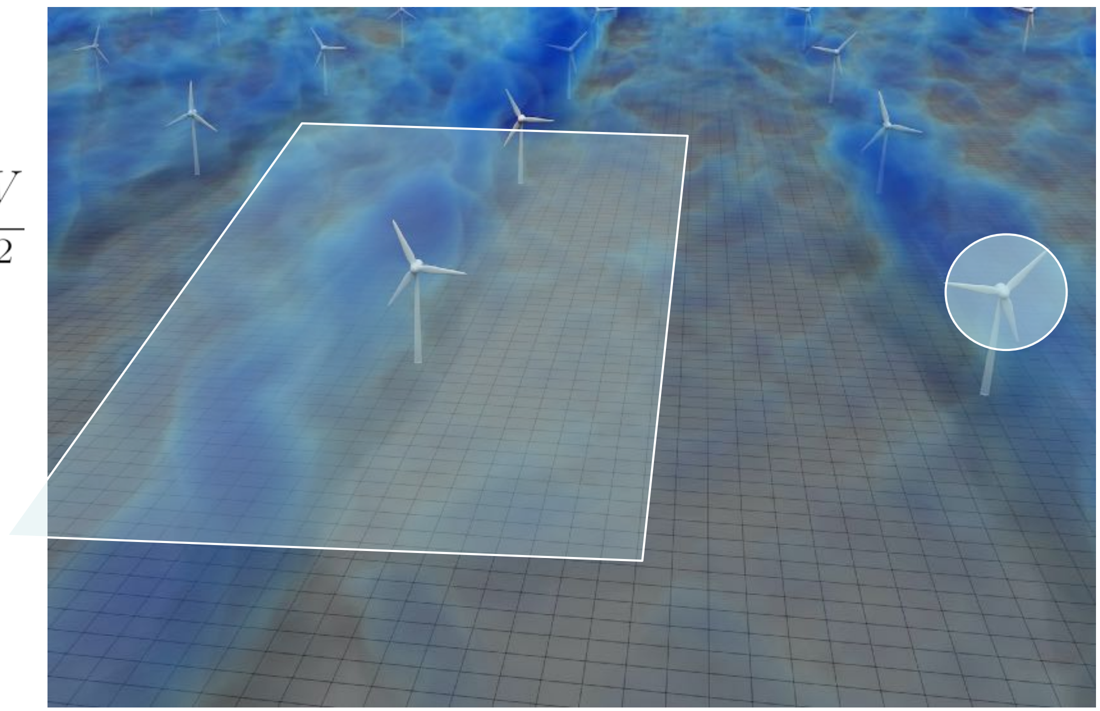
$$\frac{P}{A} = \frac{0.3 \times 5MW}{49 \times 120^2}$$

$$= 2.1 \frac{W}{m^2} \text{ or } \frac{MW}{km^2}$$



20 ~ (4^2 - 5^2)



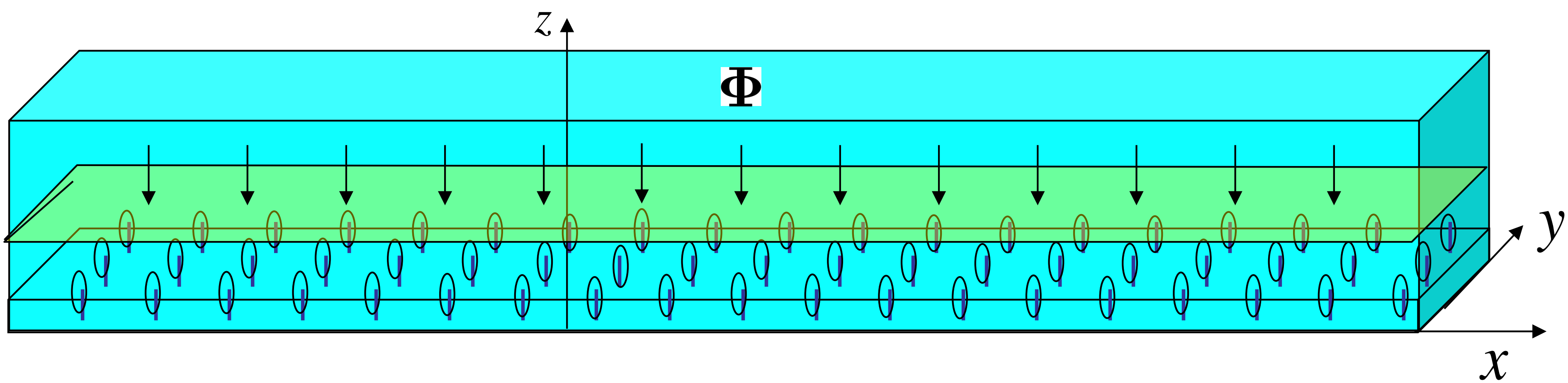
$$\frac{P}{A} = \frac{0.3 \times 5MW}{\pi 60^2}$$

$$= 130 \frac{W}{m^2}$$

Power-density of wind farms:

30,000 feet view: WTABL

Newman 1979, S. Frandsen 1992, Frandsen et al. 2006, Calaf et al 2010:



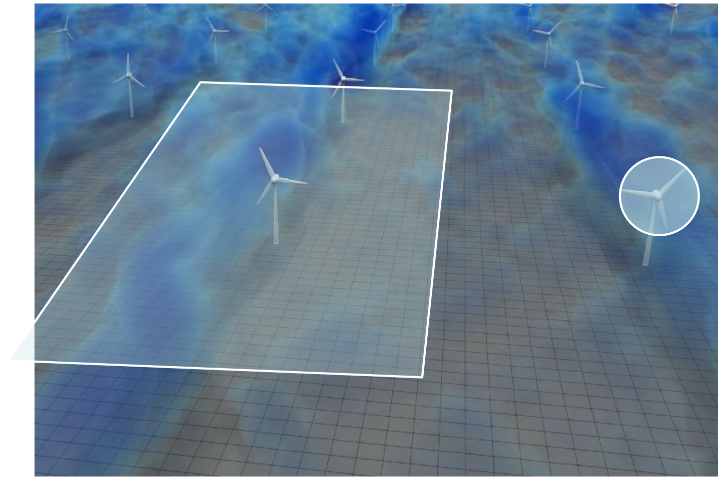
Caveats:

- Flat terrain or off-shore
- Near-neutral
- Hub-heights < 150 m or so

Power-density of wind farms:

$$\frac{P}{A} = \frac{0.3 \times 5MW}{49 \times 120^2}$$

$$= 2.1 \frac{W}{m^2} \quad \text{or} \quad \frac{MW}{km^2}$$



What is the fate of mean flow kinetic energy in large wind farms, where is it dissipated, and how is it entrained from above?

Calaf et al. PoF 2010

$$\frac{1}{2}\langle \overline{u} \rangle_{xy}^2$$

$$\frac{1}{\rho}\langle\overline{u}\rangle\frac{dp_\infty}{dx} + \frac{\partial}{\partial z}\left(\langle\overline{u'w'}\rangle\langle\overline{u}\rangle + \langle\overline{u}''\overline{w}''\rangle\langle\overline{u}\rangle\right) = \langle\overline{u'w'}\rangle\frac{\partial\langle\overline{u}\rangle}{\partial z} + \langle\overline{u}''\overline{w}''\rangle\frac{\partial\langle\overline{u}\rangle}{\partial z} + w_T$$

$$\nabla\cdot\mathbf{\Phi} \;=\; \text{“}dissipation\text{”} \;+\; \text{“}turb.power\text{”}$$

Flux of mean kinetic energy (unperturbed ABL):

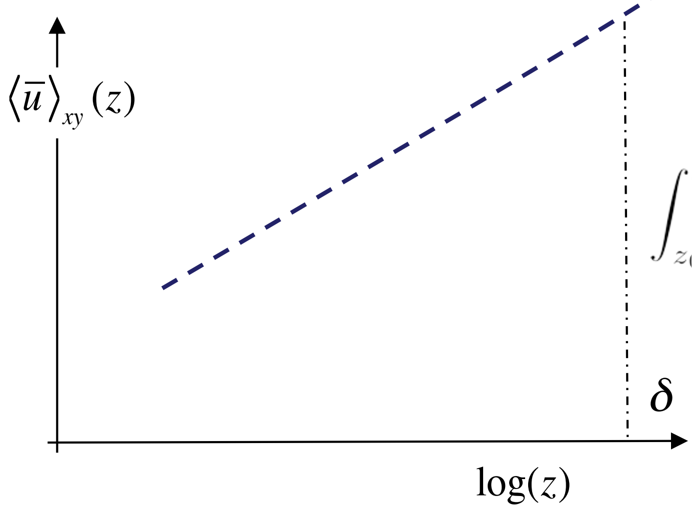
$$\rho\langle\overline{u'w'}\rangle\langle\overline{u}\rangle \approx \rho\, u_*^2\, U \approx 1.2 \times 0.43^2 \times 10 \approx 2.2\frac{W}{m^2}$$

$$U = \frac{0.43}{0.4}\ln\frac{100}{0.01} \approx 10$$

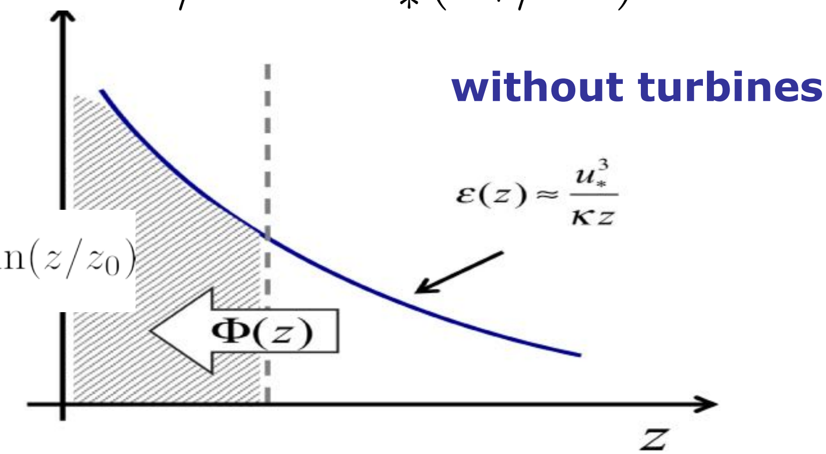
seems too low !

mean kinetic energy in the WTABL: $\frac{1}{2}\langle \overline{u} \rangle_{xy}^{2}$

$$\epsilon(z) = -\overline{u'w'}\partial\bar{u}/\partial z = u_*^2(u_*/\kappa z)$$



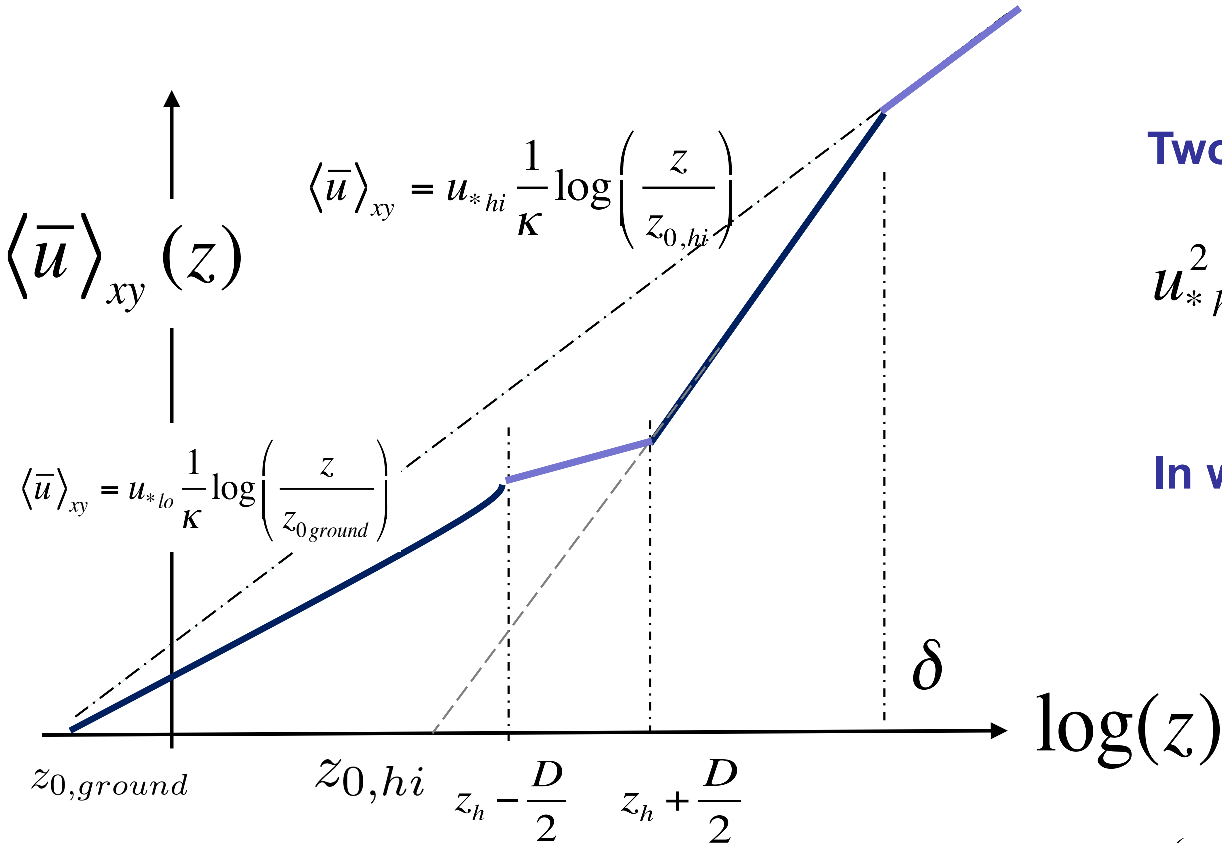
$$\int_{z_0}^{z} \epsilon(z')dz' = (u_*^3/\kappa)\ln(z/z_0)$$



$$\Phi(z) = -\overline{u'w'}\,\bar{u} = (u_*^3/\kappa)\ln(z/z_0)$$

mean kinetic energy in the WTABL: $\frac{1}{2}\langle \overline{u} \rangle_{xy}^2$

S. Frandsen 1992, Frandsen et al. 2006, Calaf et al 2010, Stevens 2015..:



Two "constant stress" layers with:

$$u_{*hi}^2 \approx u_{*lo}^2 + \frac{1}{2} C_T \frac{\pi}{4 s_x s_y} U_h^2$$

In wake layer, reduced slope:

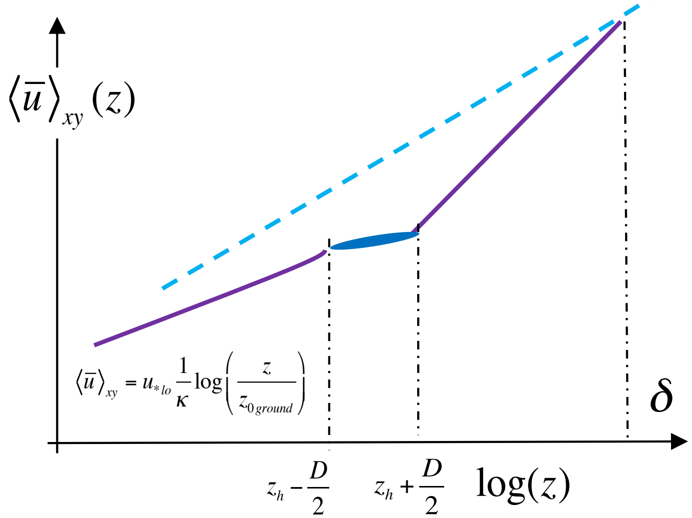
$$\frac{\partial \langle \overline{u} \rangle}{\partial z} = \frac{1}{\kappa u_* z_h + \nu_w} u_*^2$$

Effective wind farm roughness: $z_{0,hi} = z_h \left(1 + \frac{D}{2z_h}\right)^{\beta} \exp\left(-\left[\frac{\pi C_T}{8\kappa^2 s_x s_y} + \left(\ln\left[\frac{z_h}{z_{0,ground}}\left(1 - \frac{D}{2z_h}\right)^{\beta}\right]\right)^{-2}\right]^{-1/2}\right)$

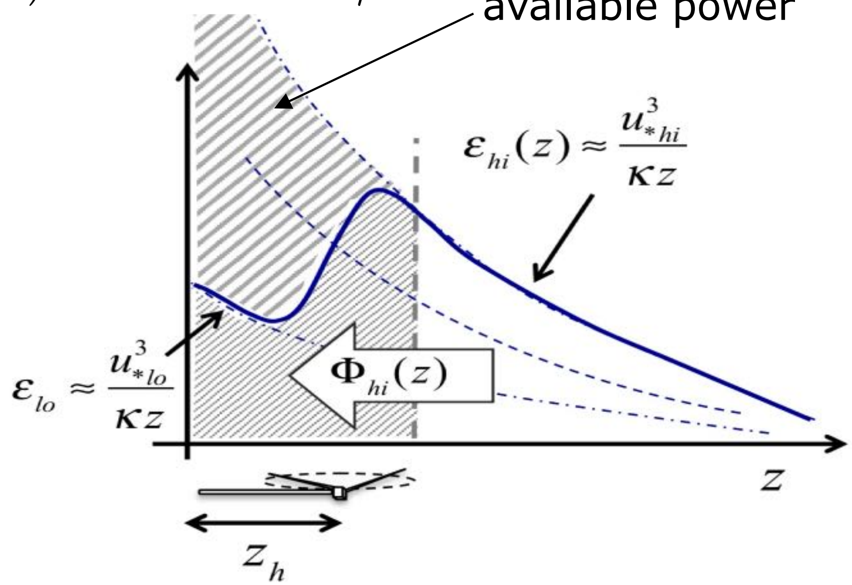
Friction velocity above wind farm: $u_{*,hi} = u_{*,\mathrm{ref}} \frac{\ln(\delta / z_{0,\mathrm{ground}})}{\ln(\delta / z_{0,hi})}$

mean kinetic energy in the WTABL: $\frac{1}{2}\langle \overline{u} \rangle_{xy}^2$

with turbines



$$\epsilon(z) = -\overline{u'w'}\,\partial \bar{u}/\partial z$$

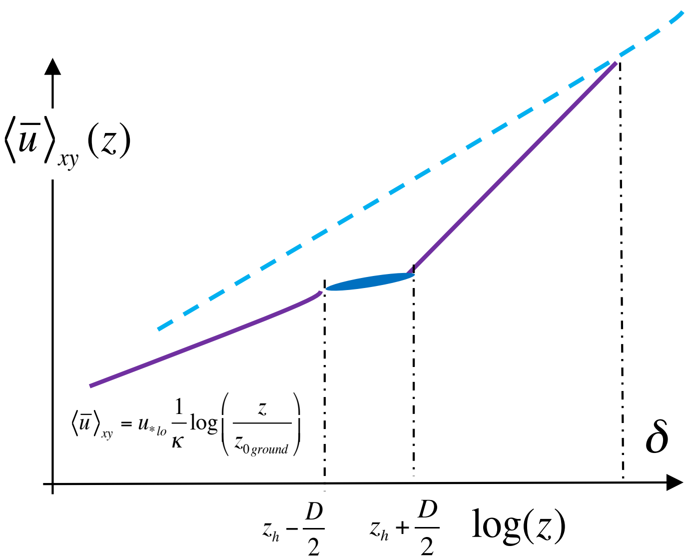


Instead of being dissipated entirely in BL, mean KE extracted by turbines and dissipated

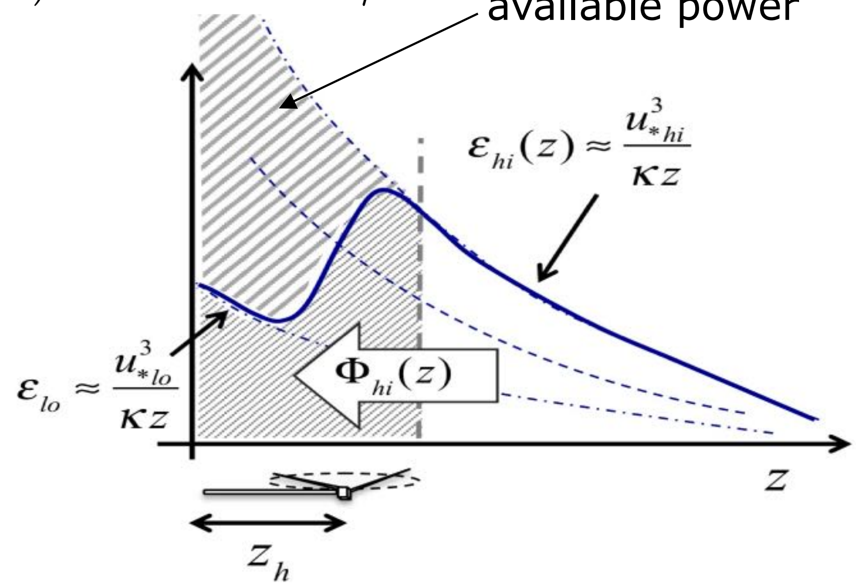
$$\Phi_{\text{hi}}(z) = -\overline{u'w'}\,\bar{u} = (u_{*,hi}^3/\kappa)\ln(z_h/z_{0,hi})$$

mean kinetic energy in the WTABL: $\frac{1}{2}\langle\overline{u}\rangle_{xy}^2$

with turbines



$$\epsilon(z) = -\overline{u'w'}\partial\bar{u}/\partial z$$



$$u_{*,hi} = u_{*,\mathrm{ref}}\frac{\ln(\delta/z_{0,\mathrm{ground}})}{\ln(\delta/z_{0,hi})} = 0.43\frac{\ln(1000/0.01)}{\ln(1000/1)} = 0.7\ m/s$$

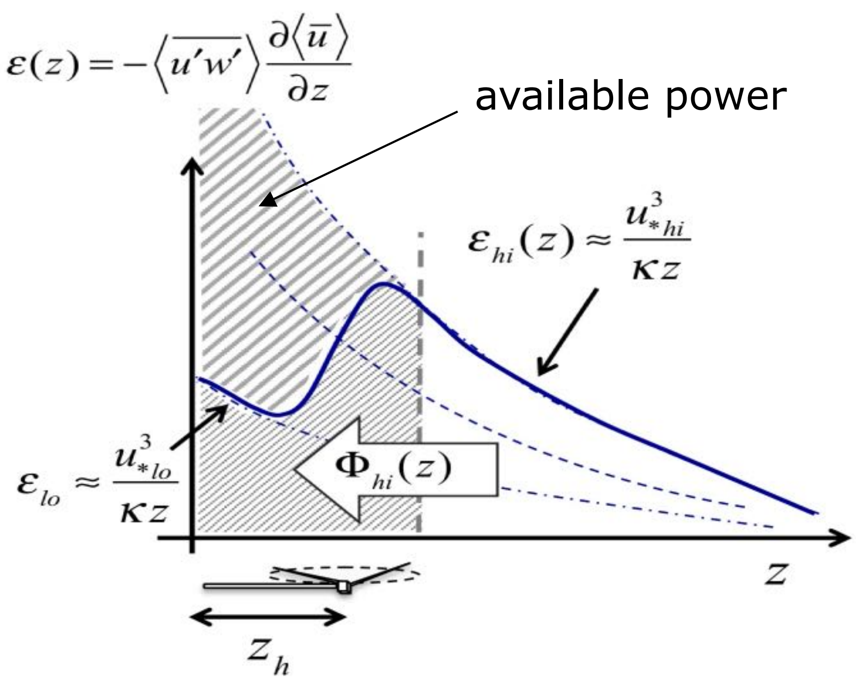
$$U = (u_{*,hi}/\kappa)\ln(z/z_{0,hi}) = \frac{0.7}{0.4}\ln(1000/1) = 8m/s$$

$$\Phi(z) = -\rho\,\overline{u'w'}\,\bar{u} = \rho\frac{u_{*,hi}^3}{\kappa}\ln(z/z_{0,hi}) = 1.2\times\frac{0.71^3}{0.4}\times\ln(100/1) = 5\,W/m^2$$

mean kinetic energy in the WTABL: $\frac{1}{2}\langle \overline{u} \rangle_{xy}^{2}$

with turbines

$$\Phi(z) = -\rho\,\overline{u'w'}\,\bar{u} = \rho\,\frac{u_{*,hi}^{3}}{\kappa}\ln(z/z_{0,hi}) = 1.2 \times \frac{0.71^{3}}{0.4} \times \ln(100/1) = 5\,W/m^{2}$$



5 W/m² →

2.5 W/m² turbine power	2.5 W/m² dissipation

A pertinent new "canonical turbulent flow": The windturbine-array boundary layer (WTABL)

(=WAKES + ATMOSPHERIC BOUNDARY LAYER)

Horns Rev 1: Photograph: Christian Steiness

Photo credit: Bel Air

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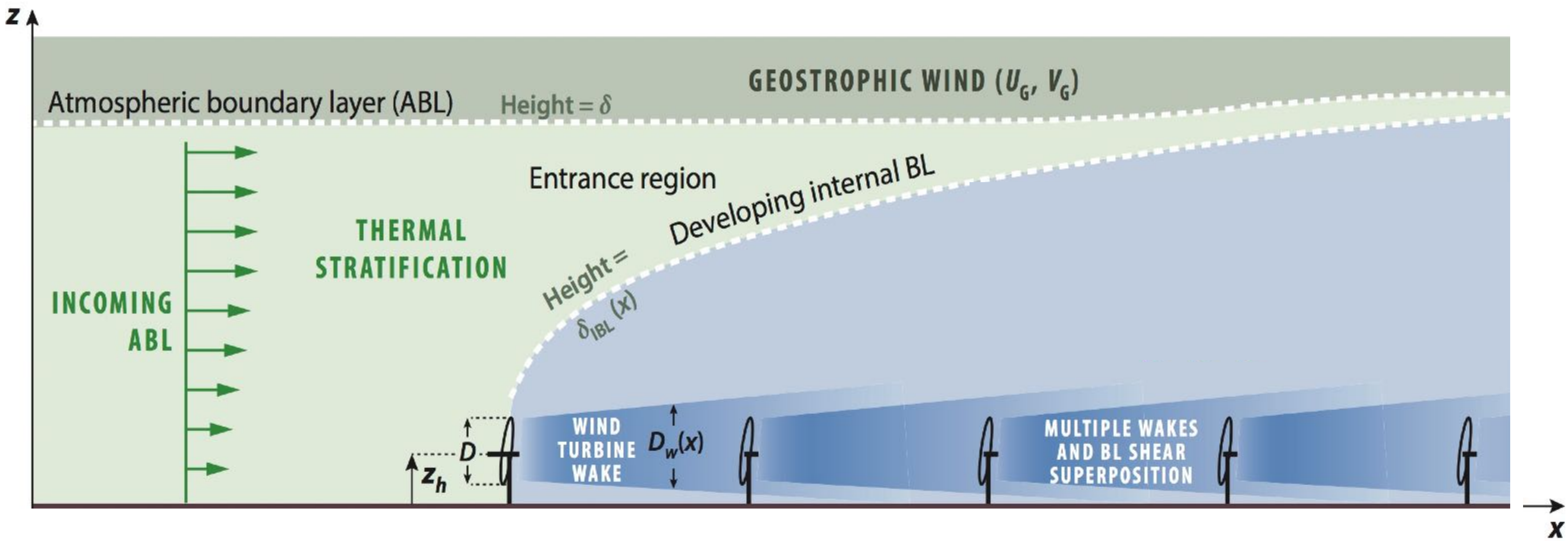
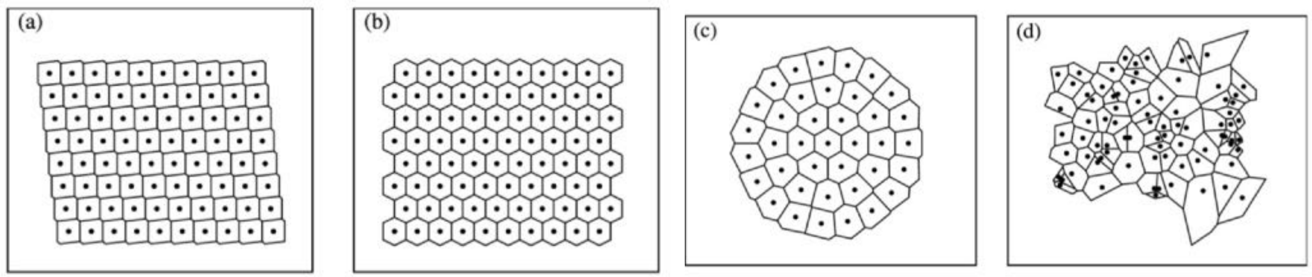
Fluid phenomena in the wind turbine-array boundary layer (WTABL)



From: R.J.A.M. Stevens & C.M., “Flow structure and turbulence in wind farms”, (2017), Annu. Rev. Fluid Mech. **49**, 311-339.

Goal of wind farm models:

- Given:
  - Inflow ABL characteristics $U_\infty(y,z;t)$ ..
  - Turbine specs (D, z_h, aerodynamic parameters…).
  - Wind farm layout



- Analytical mean velocity distribution model across entire wind farm, $\overline{u}(x,y,z;t)$

  Evaluate mean velocity across rotor "circle", U_{turb}

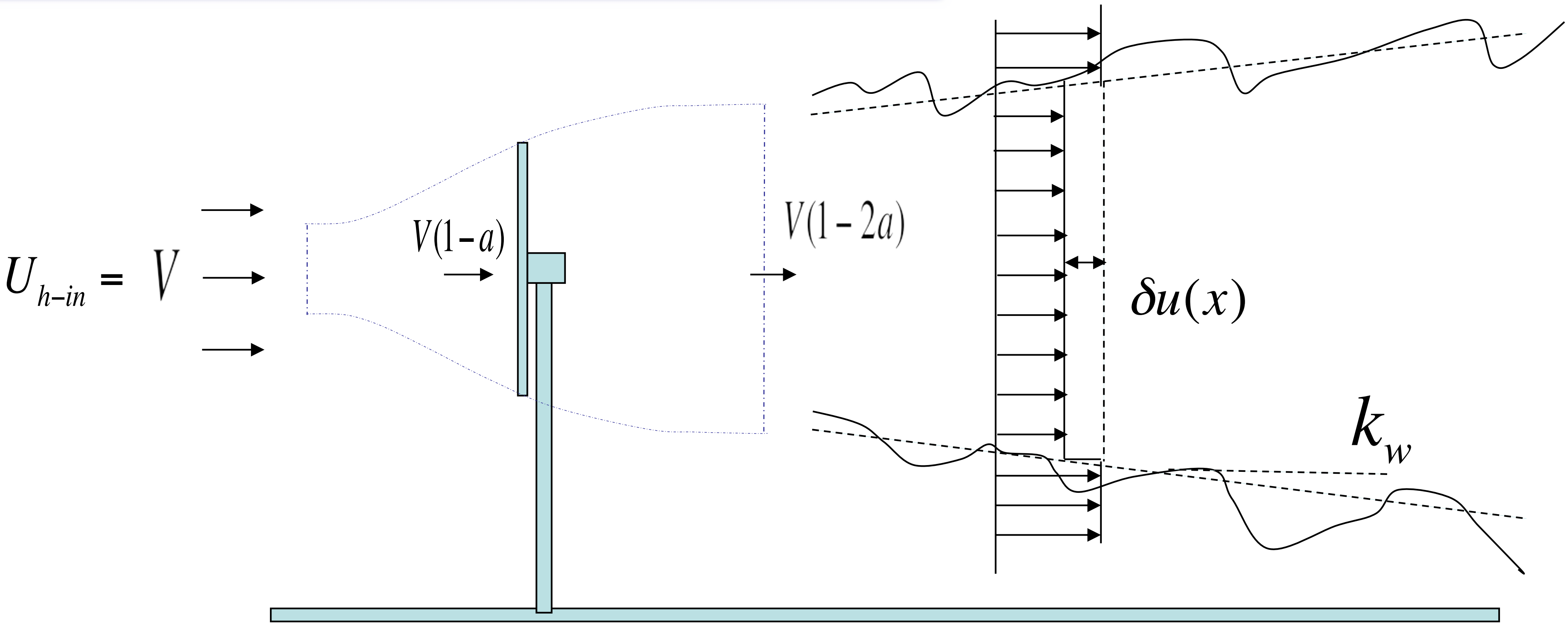
  Compute power generated by each turbine: $P_{\text{turb}} = \frac{1}{2} C_P \rho \frac{\pi}{4} D^2 \, U_{\text{turb}}^3$

- This can then be used e.g. for wind farm layout optimization .. $\max\left(\sum_{turbs} P_{turb}\right)$

- So many parameters: need analytical models (not PDE-based, no RANS/LES etc)

Jensen model: The single wake

Lissaman (1979) / Jensen (1984)



inviscid momentum theory gives "IC" for wake model

wake model: turbulence governs growth rate k_w of wake

$$a = \frac{1}{2}\left(1 - \sqrt{1 - C_T}\right)$$

$$\delta u(x, j) = U_{h0} - u(x) = \frac{2aU_{h0}}{\left(1 + 2k_w \dfrac{x - x_j}{D}\right)^2}$$

Recent improvements:

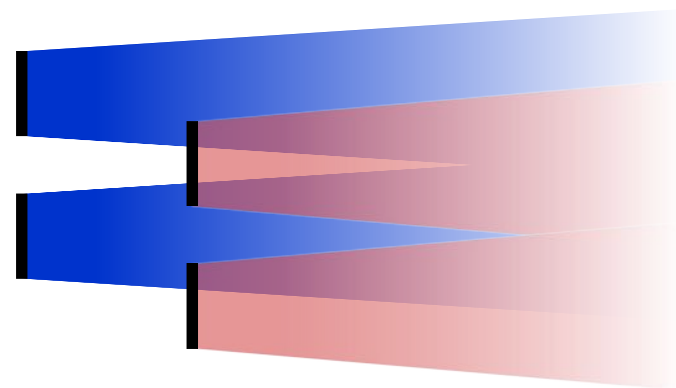
Bastankah & Porté-Agel, Ren Energy (2014)
Shapiro et al. Energies (2019)

Linear superposition: $\bar{u}\left(x,y,z;t\right)=U_{\infty}\left(y,z;t\right)-\sum_{n}\delta u_{n}(x)\,W_{n}\left(x,y,z\right)$

Velocity deficit (RANS), single wake

$$\bar{u}(x,r)=U_{\infty}-\delta u(x)W\left(\frac{r}{\ell(x)}\right),$$

$$U_{\infty}\frac{\partial\delta u}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\nu_{T}(x)\frac{\partial\delta u}{\partial r}\right),\qquad \xi=\frac{r}{\ell}$$



Eddy viscosity: Velocity scale is friction velocity: $\upsilon=u_{*}$

$$\nu_{T}(x)=u_{*}\ell(x).$$

$$\frac{U_{\infty}}{u_{*}}\frac{\ell(x)}{\delta u(x)}\frac{d\delta u}{dx}W=\left(\underbrace{\frac{U_{\infty}}{u_{*}}\frac{d\ell}{dx}}\xi+\frac{1}{\xi}\right)W'+W''.$$

Wake grows linearly: $\ell(x)\sim\frac{u_{*}}{U_{\infty}}x$

Actuator disk theory:

$$\delta u_{0}=U_{\infty}\left(1-\sqrt{1-C_{T}}\right)=\frac{1}{2}C'_{T}u_{d}$$

$$\delta u\ (x)=\frac{\delta u_{0}}{2\left(d_{w}(x)\right)^{2}}\left[1+erf\left(\frac{x}{R\sqrt{2}}\right)\right]$$

$$d_{w}(x)=1+2k_{w}\frac{x}{D}$$

Recent improvements:

Linear superposition:

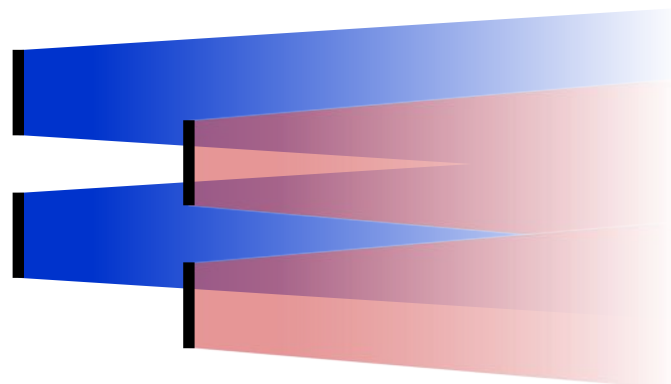
$$\bar{u}(x,y,z;t) = U_\infty(y,z;t) - \sum_n \delta u_n(x,t)\, W_n(x,y,z)$$

More realistic wake function:

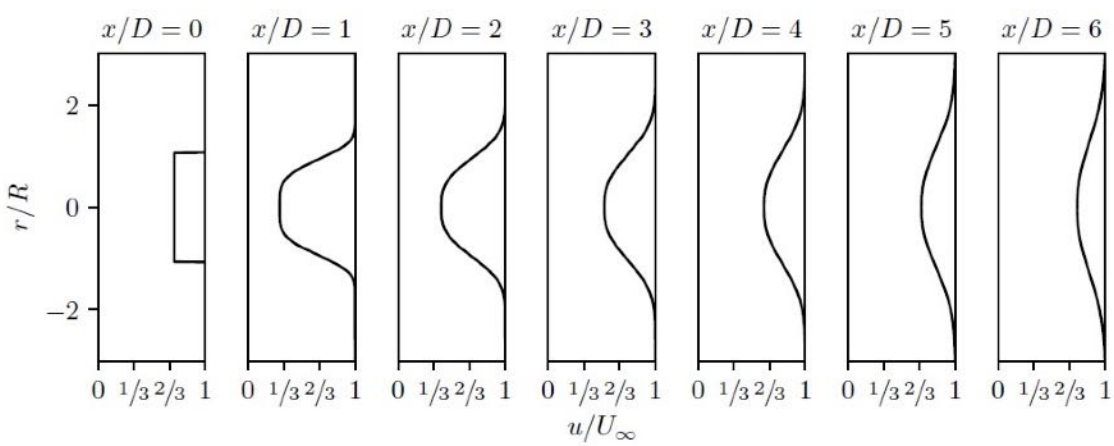
Variable supergaussian

Bastankah & Porté-Agel, Ren Energy (2014)

Shapiro et al. Energies (2019)



$$W(x,r) = C(x)\exp\left(\frac{-D^2}{8\sigma_0^2}\left(\frac{2r}{Dd_w(x)}\right)^{p(x)}\right)$$

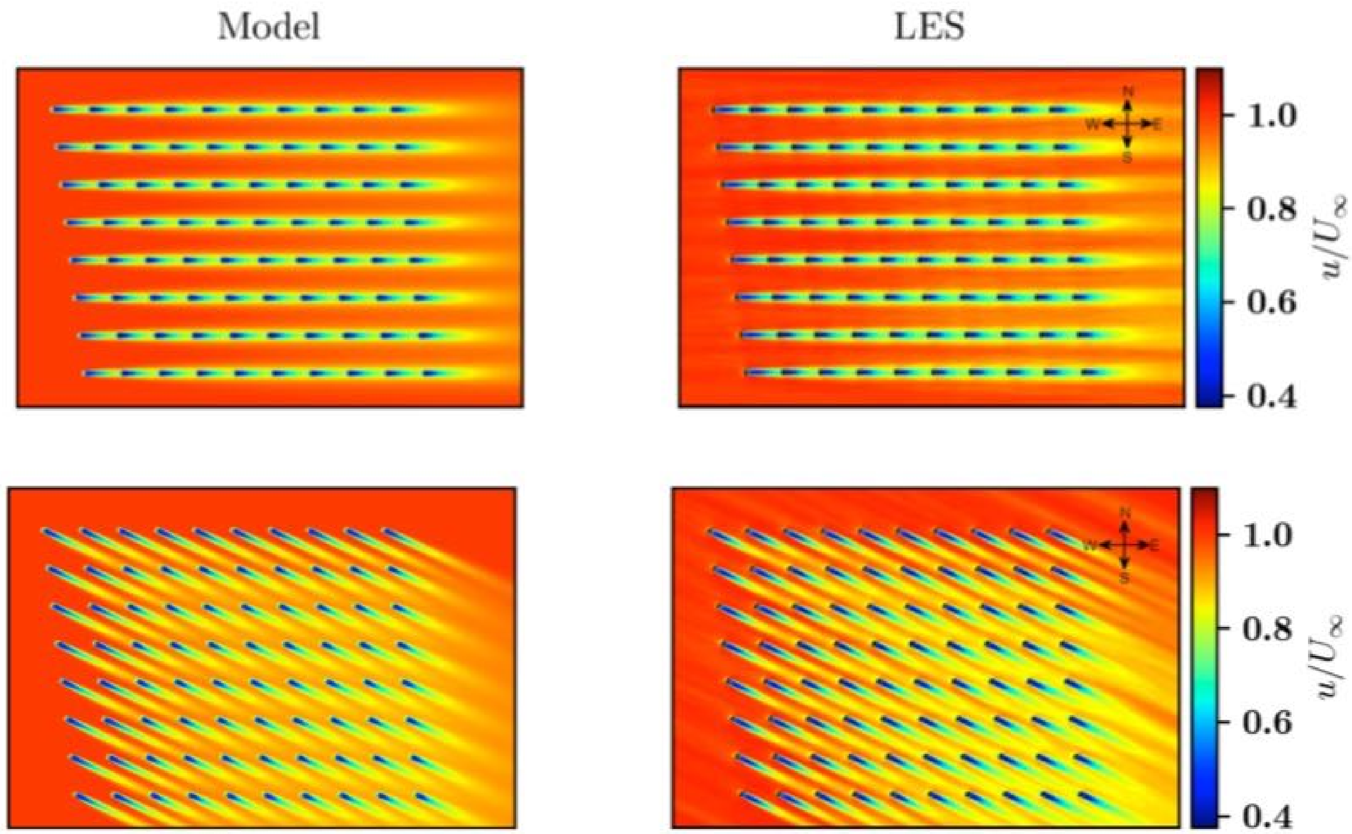


Velocity deficit

$$\delta u_n(x) = \frac{\delta u_{0,n}}{2\left(d_{w,n}(x)\right)^2}\left[1 + erf\left(\frac{x}{R\sqrt{2}}\right)\right]$$

Recent improvements:

Linear superposition: $\overline{u}\left(x,y,z;t\right)=U_{\infty}\left(y,z;t\right)-\sum_{n}\delta u_{n}(x,t)\,W_{n}\left(x,y,z\right)$

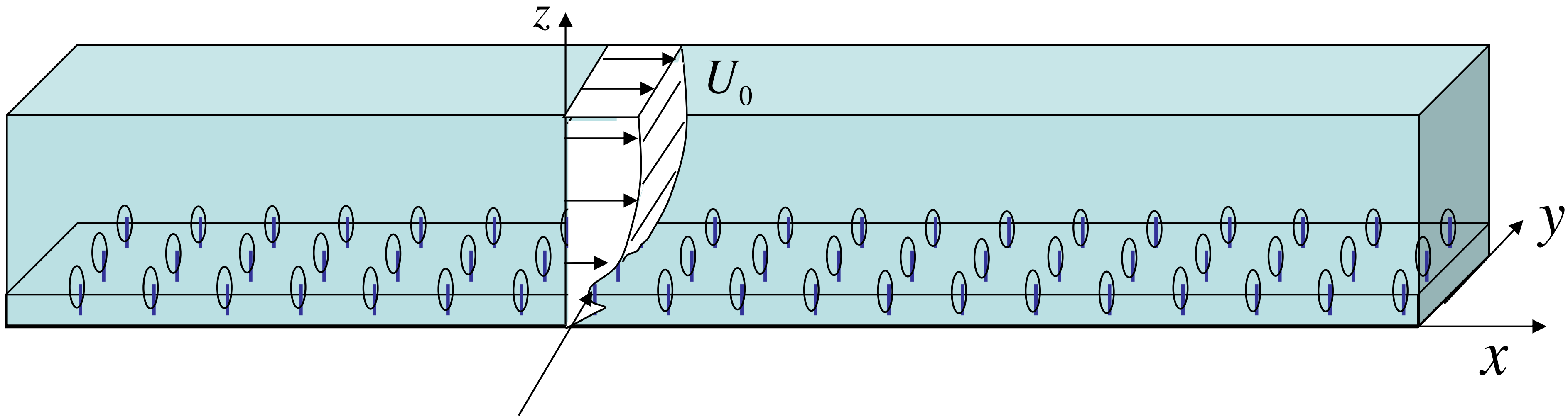


Shapiro et al. Energies (2019)   Porté-Agel et al. Energies (2013)

Remaining challenges:

the **deep array** effect - limiting behavior for very large wind farms

A boundary layer (canopy flow) view: the mean velocity vertical profiles in fully developed WTABL



$$U(z) = \langle \overline{u}(x,y,z) \rangle_{xy}$$

horizontal (canopy) average

Data: from LES of WTABL typical simulation setup:

- LES code: horizontal pseudo-spectral (periodic B.C.), vertical: centered 2nd order FD (Moeng 1984, Albertson & Parlange 1999, Porté-Agel et al. 2000, Bou-Zeid et al. 2005)

$$\nabla \cdot \tilde{\mathbf{u}} = 0$$

$$\frac{\partial \tilde{\mathbf{u}}}{\partial t} + \tilde{\mathbf{u}} \cdot \nabla \tilde{\mathbf{u}} = -\nabla \tilde{p}^* - \nabla \cdot \boldsymbol{\tau} + \mathbf{f}$$

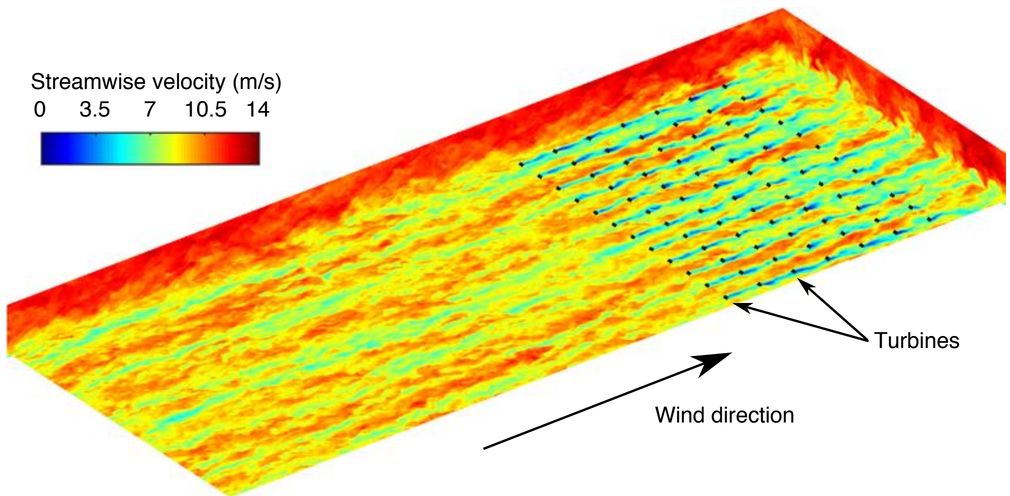
High-fidelity, but has large computational cost

Actuator disk forcing:

$$F = -\frac{1}{2}\rho A C_T' \langle u \rangle_d^2$$

Power:

$$P = \frac{1}{2}\rho A C_T' \langle u \rangle_d^3$$



$$H = 1000 - 1500m, \quad L_x = \pi H - 2\pi H, \quad L_y = \pi H$$

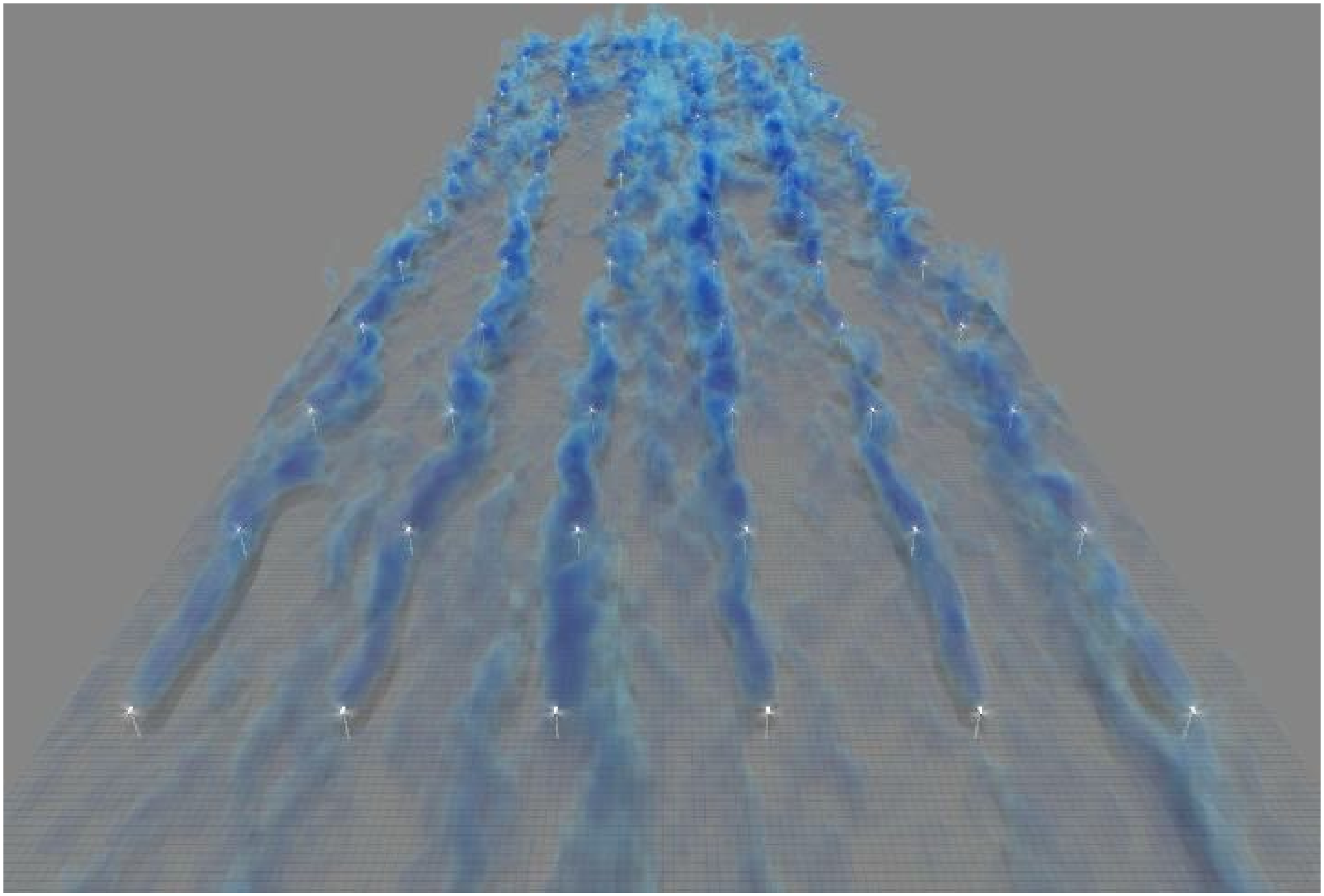
$$(N_x \times N_y \times N_z) = 128 \times 128 \times 128 \quad \rightarrow \quad 1024 \times 512 \times 512$$

- Horizontal periodic boundary conditions (for FD-WTABL, or precursor for developing)
- Top surface: zero stress, zero w
- Bottom surface B.C.: w=0 + Wall stress: Standard wall function relating wall stress to $u(z_1)$
- Scale-dependent dynamic Lagrangian model eddy-viscosity (*no* adjustable parameters
- More details: Calaf et al. Phys. Fluids. **22** (2010) 015110

Large Eddy Simulations of large wind farms (R. Stevens)

JHU-LES codeVisualization courtesy of D. Brock (Extended Services XSEDE)

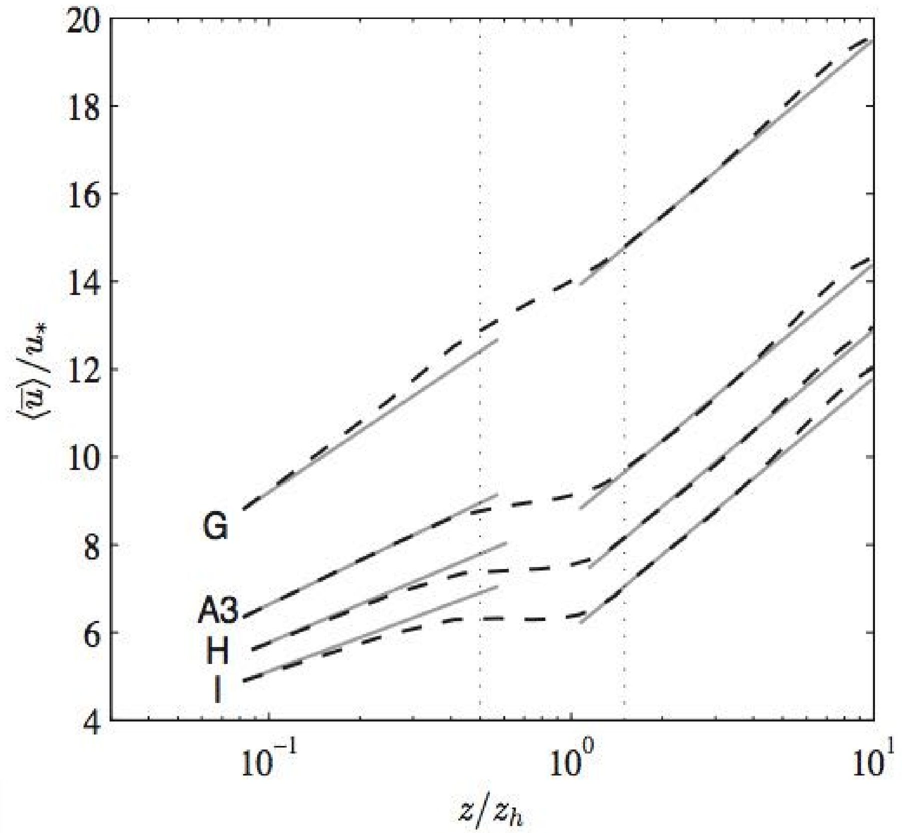
$$H = 1500m, \quad L_x = 8\pi H, \quad L_y = 3\pi H \qquad (N_x \times N_y \times N_z) = 1024 \times 512 \times 512$$

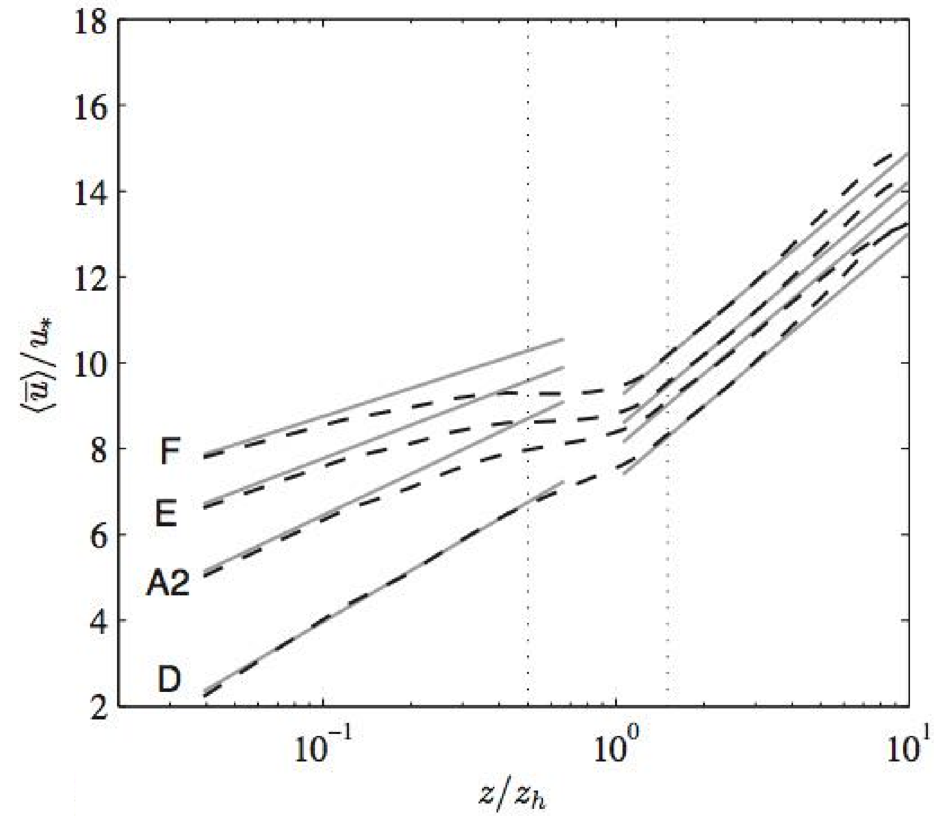


Simulations: R.J.A.M. Stevens et al, JRSE **6**, 023105 (2014), using ADM in JHU-LES code (turbine rotation only for visualization). White particles: passive tracers

Horizontal mean velocity in FD-WTABL from LES (ADM):

Calaf et al 2010 (confirming hypothesis by Frandsen 1992): **2 log-laws**



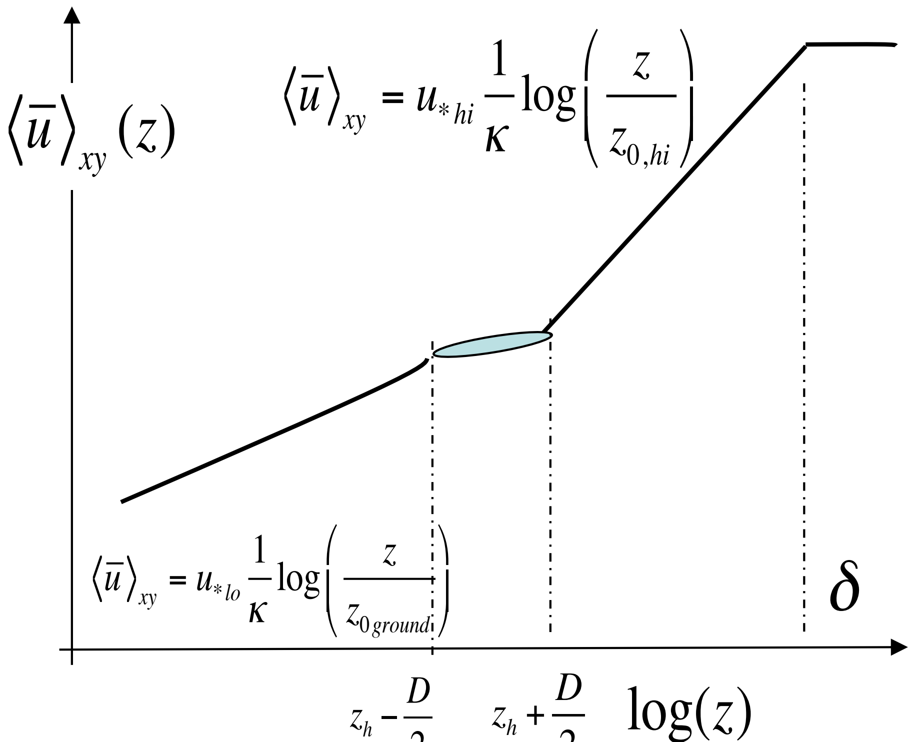


Other studies of WTABL velocity distributions:

- Cal et al. (JRSE 2010)
- Johnstone & Coleman (J Wind Eng & Ind A, 2012)
- Yang, Kang & Sotiropoulos (PoF 2012)
- Chamorro & Porté-Agel (2013)
- Chatterjee & Peet (Pys Rev Fluids 2018)
- Ghate & Lele (J Fluid Mech, 2017)

Top down model:

S. Frandsen 1992, Frandsen et al. 2006, Calaf et al 2010, Stevens 2015:



Two "constant stress" layers with:

$$u_{*,hi}^2 = u_{*,lo}^2 + \frac{1}{2} c_{ft} \overline{u}_h^2 \qquad c_{ft} = C_T \frac{\frac{\pi}{4} D^2}{A_{pf}}$$

In wake layer, reduced slope:

$$\frac{\partial \langle \overline{u} \rangle}{\partial z} = \frac{1}{\kappa u_* z_h + \nu_w} u_*^2$$

Effective wind farm roughness:

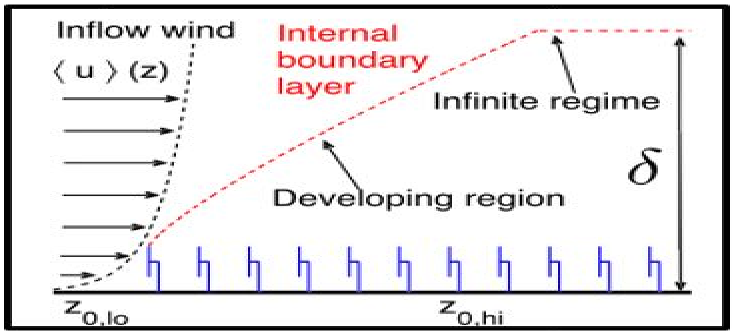
$$z_{0,hi} = z_h \left(1 + \frac{D}{2z_h}\right)^{\beta} \exp\left(-\left[\frac{c_{ft}}{2\kappa^2} + \left(\ln\left[\frac{z_h}{z_{0,ground}} \left(1 - \frac{D}{2z_h}\right)^{\beta} \right] \right)^{-2} \right]^{-1/2} \right)$$

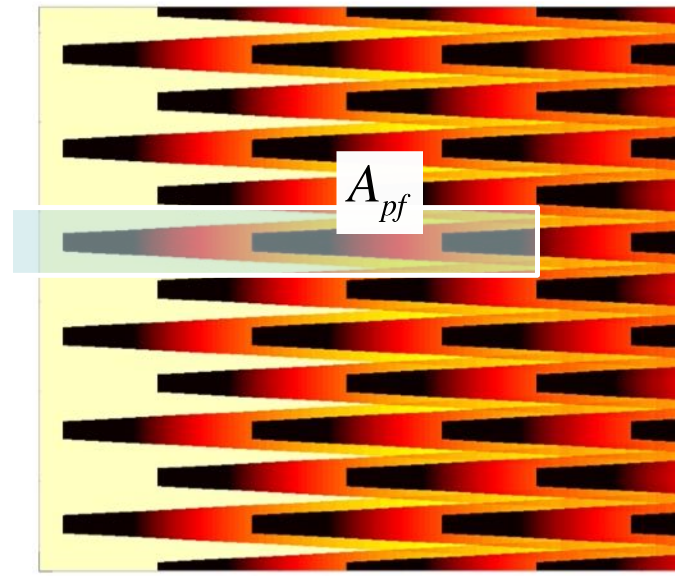
Mean velocity at hub height, normalized by ABL unperturbed inflow:

$$\frac{\overline{u}_h^{td}}{U_{h\infty}} = \frac{\ln(\delta / z_{0,\mathrm{lo}})}{\ln(\delta / z_{0,\mathrm{hi}})} \ln\left[\left(\frac{z_\mathrm{h}}{z_{0,\mathrm{hi}}} \right) \left(1 + \frac{D}{2z_\mathrm{h}} \right)^{\beta} \right] \left[\ln\left(\frac{z_h}{z_{0,\mathrm{lo}}} \right) \right]^{-1}$$

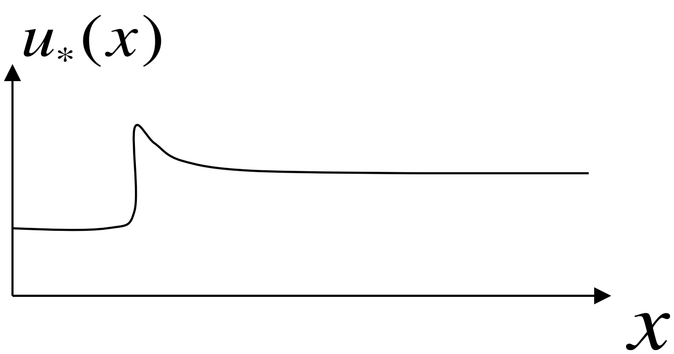
Two views: top down and bottom up

$$k_w = \frac{u_*}{U} = \frac{\kappa}{\log(z_h / z_0)}$$





Smooth-to-rough transition:

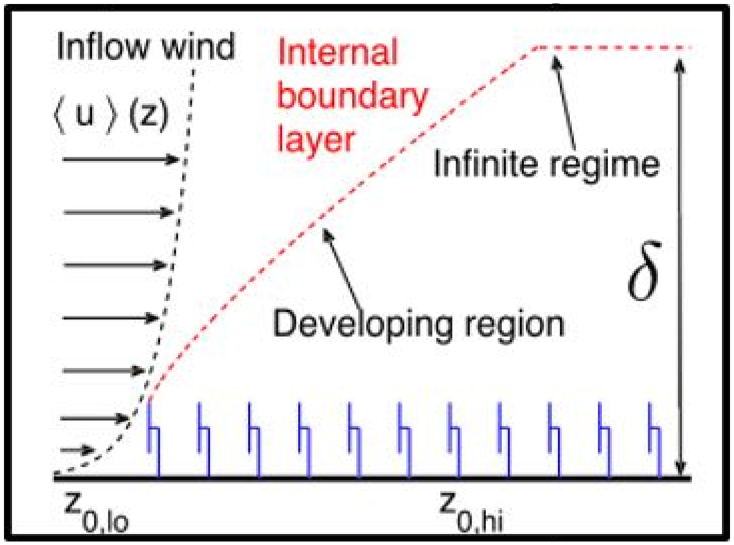


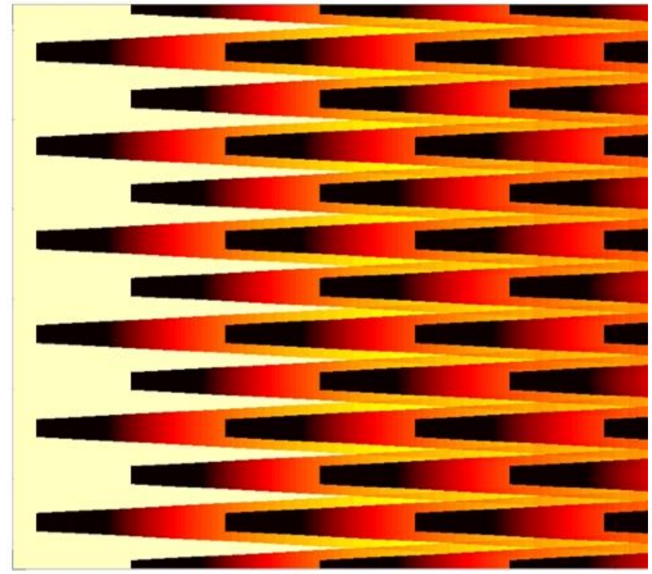
$$z_{0,hi} = z_h \left(1 + \frac{D}{2z_h}\right)^{\beta} \exp\left(-\left[\frac{c_{ft}}{2\kappa^2} + \left(\ln\left[\frac{z_h}{z_{0,ground}} \left(1 - \frac{D}{2z_h}\right)^{\beta} \right] \right)^{-2} \right]^{-1/2} \right)$$

$$u_{*,hi}(x) = u_{*,n} \frac{\ln\left(\delta_{ibl}(x) / z_{0,lo}\right)}{\ln\left(\delta_{ibl}(x) / z_{0,hi}\right)}$$

$$c_{ft,n} = \frac{\pi R^2 \sum_{i \in line} C'_{T,i} \left(u_{d,i}^{wm}\right)^2}{\sum_{i \in line} A_{pf,i} \left(\overline{u}_{h,i}^{wm}\right)^2}$$

Two views: top down and bottom up



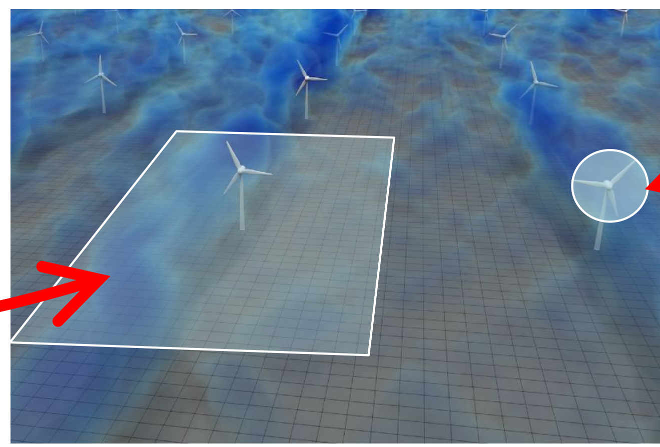


$$\bar{u}_h^{td} = U_\infty(y) \frac{\ln\left(\delta(x)/z_{0,\mathrm{lo}}\right)}{\ln\left(\delta(x)/z_{0,\mathrm{hi}}\right)} \ln\left[\left(\frac{z_h}{z_{0,\mathrm{hi}}}\right)\left(1+\frac{D}{2z_h}\right)^{\beta}\right]\left[\ln\left(\frac{z_h}{z_{0,\mathrm{lo}}}\right)\right]^{-1}$$

$$u_{d,n}^{wm} = \int_0^R \left[U_\infty(y) - \sum_{m\neq n} \delta u_m(s_x) W_m(s_x, r)\right] 2\pi r\, dr$$

Planform mean velocity

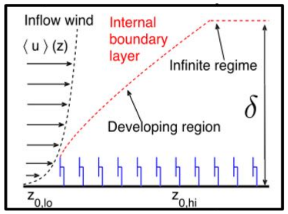
Rotor disk mean velocity

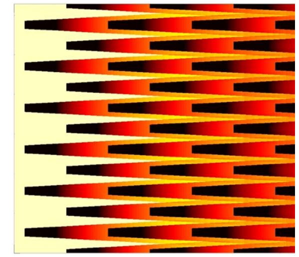


Challenge:
how to combine both views and determine k_w?

$$k_w = \frac{u_*}{U} = \frac{\kappa}{\log(z_h / z_0)}$$

Two views: top down and bottom up





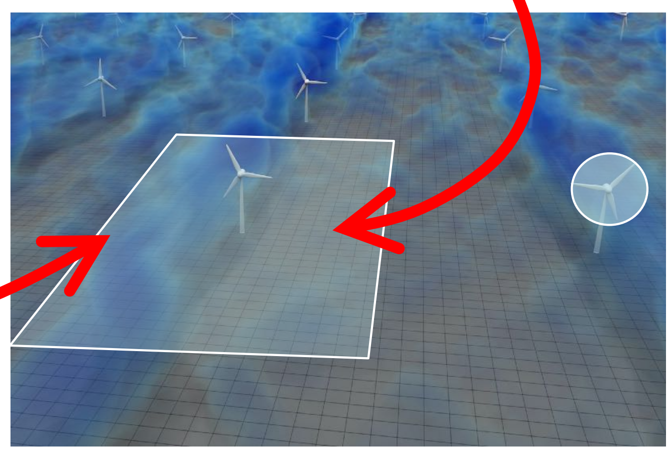
$$\bar{u}_h^{td} = U_\infty(y)\,\frac{\ln\left(\delta(x)/z_{0,\mathrm{lo}}\right)}{\ln\left(\delta(x)/z_{0,\mathrm{hi}}\right)}\,\ln\left[\left(\frac{z_\mathrm{h}}{z_{0,\mathrm{hi}}}\right)\left(1+\frac{D}{2z_\mathrm{h}}\right)^{\beta}\right]\left[\ln\left(\frac{z_h}{z_{0,\mathrm{lo}}}\right)\right]^{-1}$$

$$u_{d,n}^{wm} = \int_0^R \left[U_\infty(y) - \sum_{m\neq n}\delta u_m(s_x)W_m(s_x,r)\right]2\pi r\,dr$$

Planform mean velocity

$$\bar{u}_{h,n}^{wm} = \int_{A_{pf,n}} \left(U_\infty(y) - \sum_{m\neq n}\delta u_m(s_x)W_m(s_x,r)\right)dxdy$$

Planform mean velocity from wake model



$$k_w = \alpha\,\frac{\frac{1}{2}[u_{*hi}(x)+u_{*lo}(x)]}{\bar{u}_h^{td}}$$

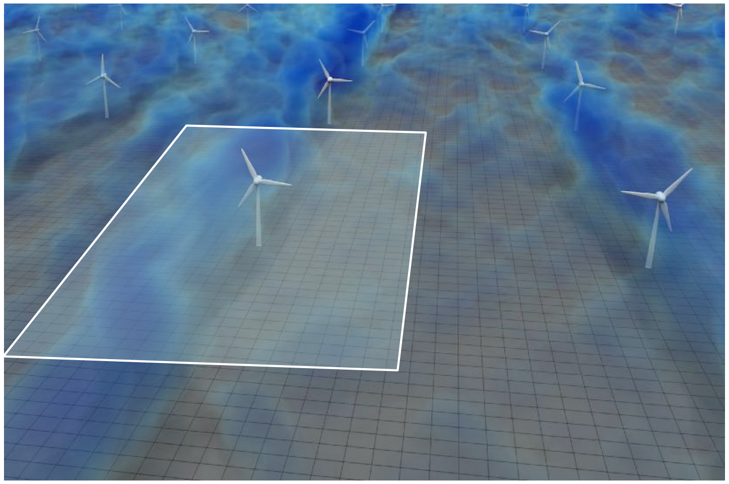
Two views: top down and bottom up

Determine α that minimizes difference between top-down and wake model plane averaged velocity

$$\min_{\alpha} \sum_{n} \left(\overline{u}_{h,n}^{wm} - \overline{u}_{h,n}^{td} \right)^2$$

Planform mean velocity from wake model

Planform mean velocity From top-down BL model

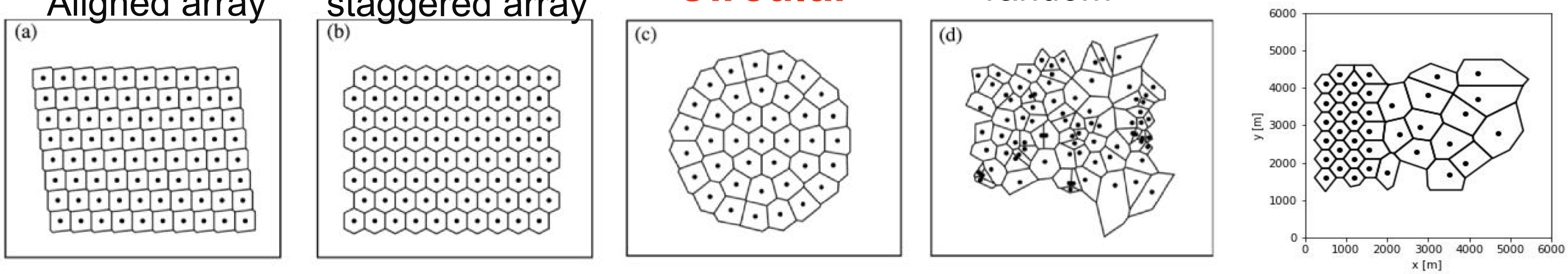


$$k_w = \alpha \frac{\frac{1}{2}[u_{*_{hi}}(x) + u_{*_{lo}}(x)]}{\overline{u}_h^{td}}$$

Comments: the Area Localized Model (ALC)

- **Voronoi tessellation to assign planform area to each turbine for arbitrary arrangements:**

Aligned array | staggered array | Circular | random | Hybrid array/random



- **For every cell (every turbine) the top-down and wake model values are calculated**
- **Uses the friction velocities from the top-down model to find the wake expansion coefficient in the wake model**

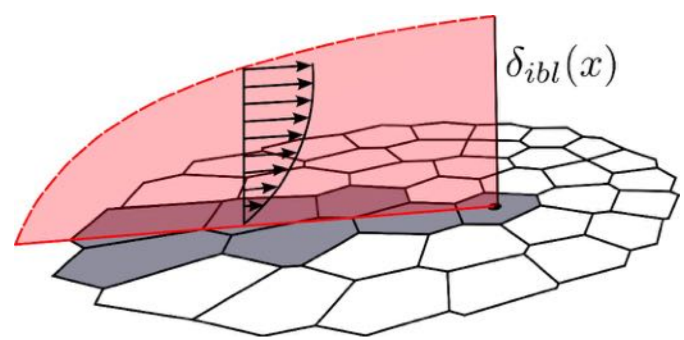
$$k_w = \alpha \frac{\frac{1}{2}[u_{*hi}(x) + u_{*lo}(x)]}{\overline{u}_h^{td}}$$

- **Uses the wake model to find the thrust coefficient in the top-down model**

$$c_{ft,n} = \frac{\pi R^2 \sum_{i \in line} C'_{T,i} \left(u_{d,i}^{wm}\right)^2}{\sum_{i \in line} A_{pf,i} \left(\overline{u}_{h,i}^{wm}\right)^2}$$

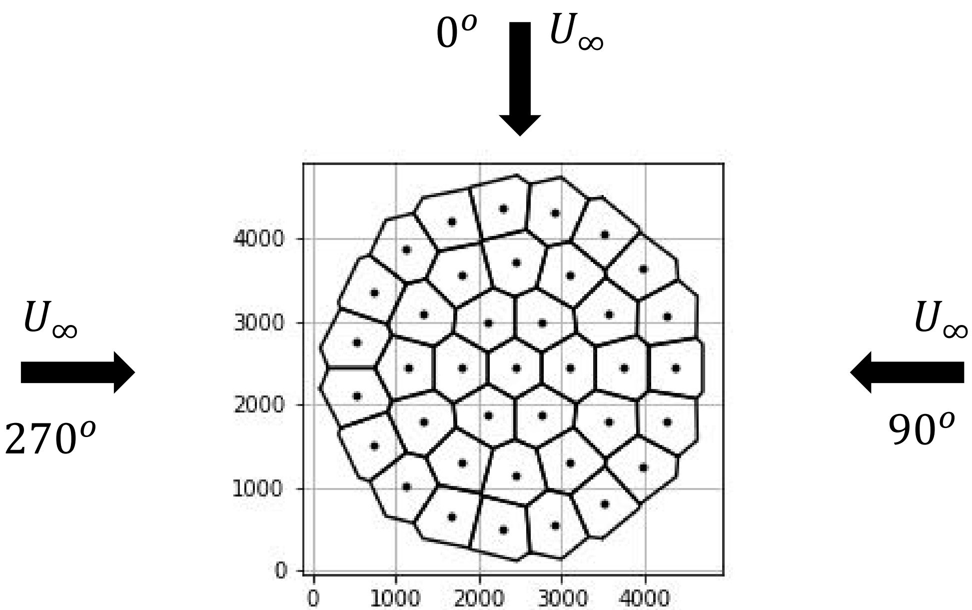
$A_{pf,n}$: planform area of the Voronoi cell for the n^{th} turbine

This also provides $z_{0,hi}$

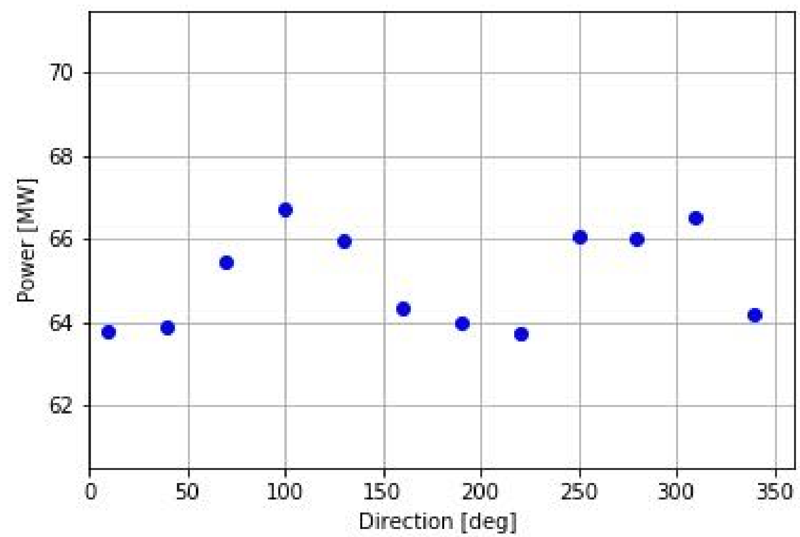


Applications: Circular wind farm:

- Using LES data provided from SOWFA solver for a circular wind farm from the National Renewable Energy Laboratory (NREL). Act line model
- Running on a circular farm:



LES NREL data:
every 30°



NREL 5 MW Reference Turbine:

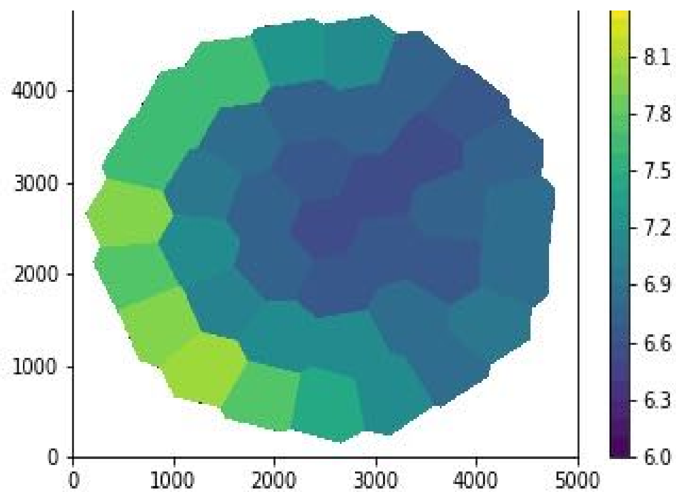
- **Diameter:** $D = 126\ m$
- **Hub Height:** $z_h = 90\ m$

Flow Conditions:

- **Lower roughness:** $z_0 = 0.15\ m$
- **Inversion layer height:** $\delta_{max} = 750\ m$

Applications: Circular wind farm:

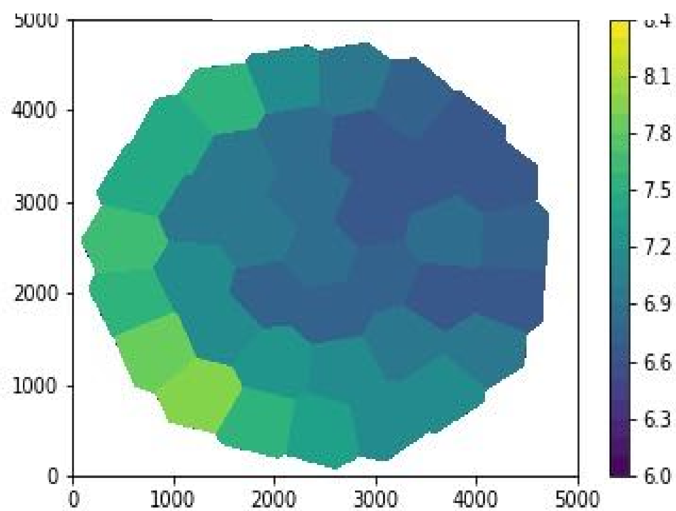
Top-down Model



$$\min_{\alpha} \sum_{n \in D} \left(\overline{u}^{wm}_{h,n} - \overline{u}^{td}_{h,n} \right)^2$$

$$k_w = \alpha \frac{\frac{1}{2}[u_{*hi}(x) + u_{*lo}(x)]}{\overline{u}^{td}_h}$$

Wake Model

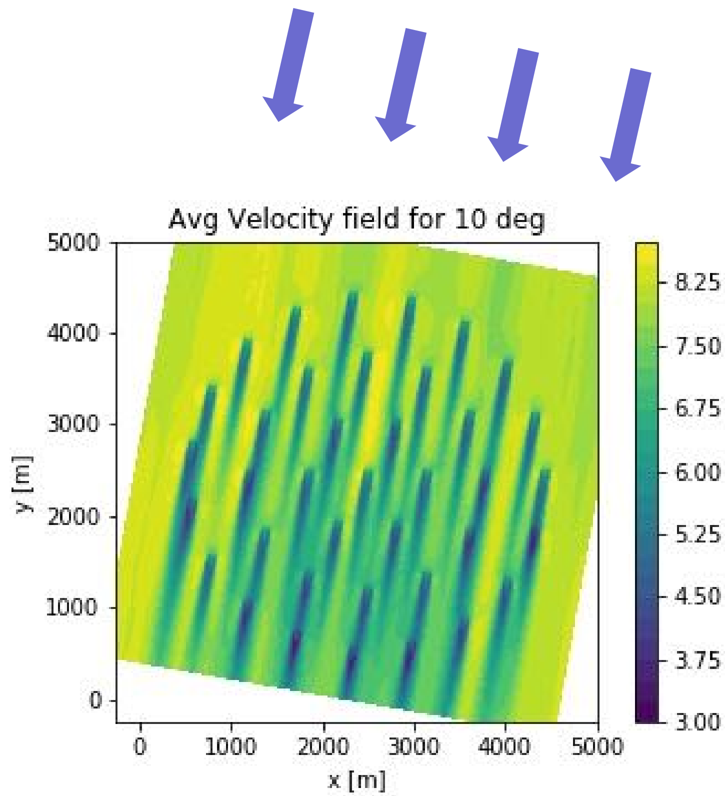


$$c_{ft,n} = \frac{\pi R^2 \sum_{i \in line} C'_{T,i} \left(u^{wm}_{d,i} \right)^2}{\sum_{i \in line} A_{pf,i} \left(\overline{u}^{wm}_{h,i} \right)^2}$$

$Power_{farm}$

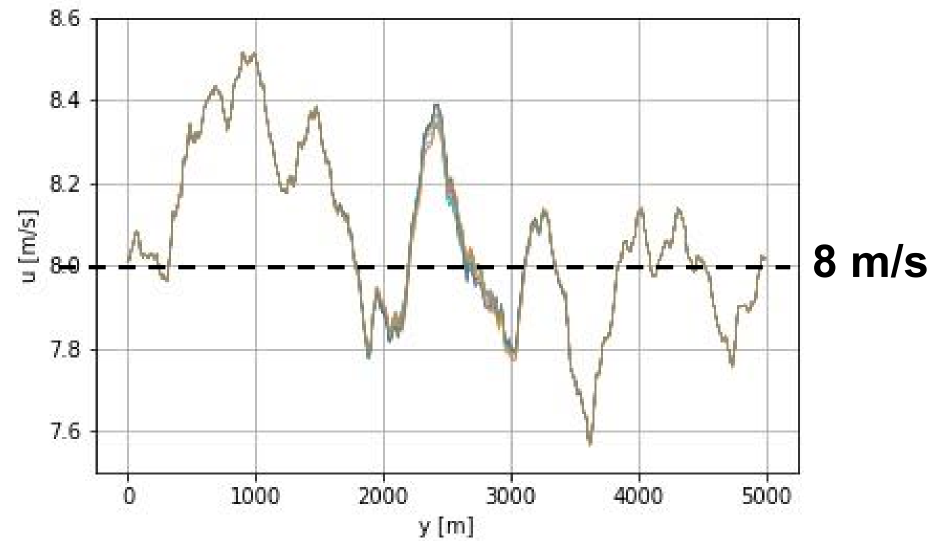
$$P_n = \frac{1}{2} \rho \pi R^2 C_T' u^3_{d,n}$$

Circular wind farm: Mean LES inflow has spanwise variations (low-high-sped streaks in ABL)



LES: Average hub-height streamwise velocity field for 10 degree direction

Velocity inflow profile for all directions (from LES)



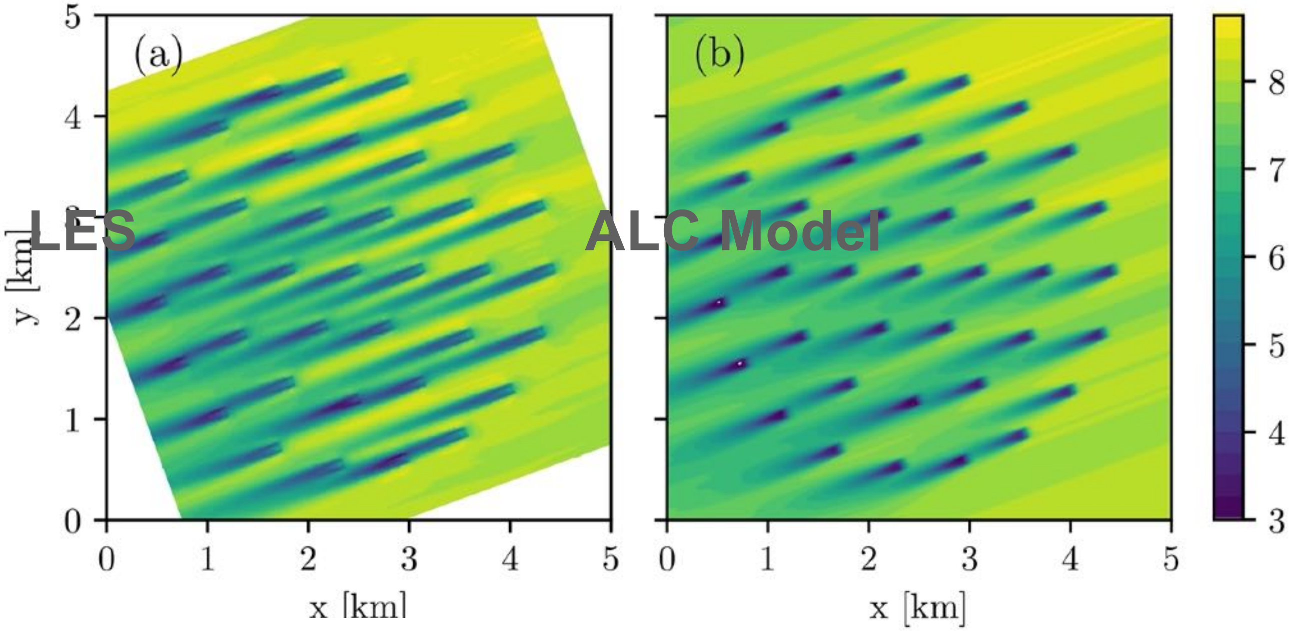
Accounted for in ALC model:

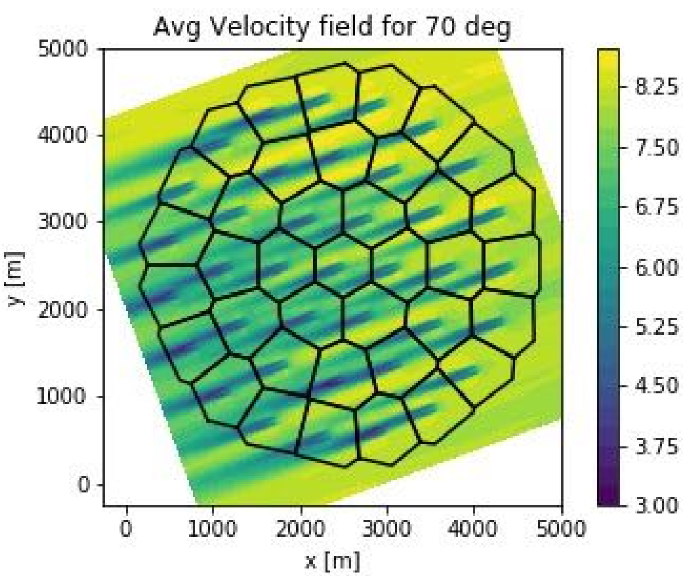
$$u_{d,n}^{wm} = \int_0^R \left[U_\infty(y') - \sum_{m \neq n} \delta u_m(s_x) W_m(s_x, r) \right] 2\pi r \, dr$$

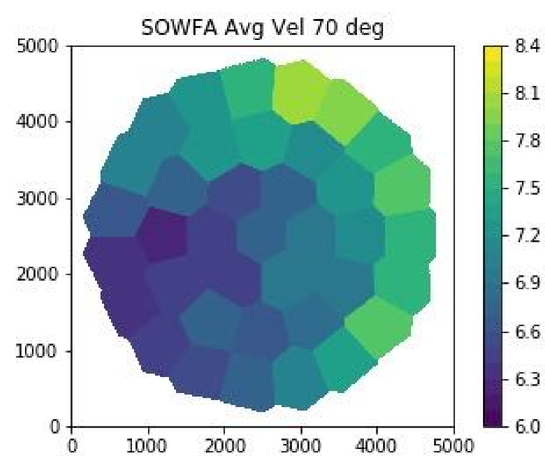
- **Comparison for 70 degrees**

LES | **ALC Model**

LES ALC Model

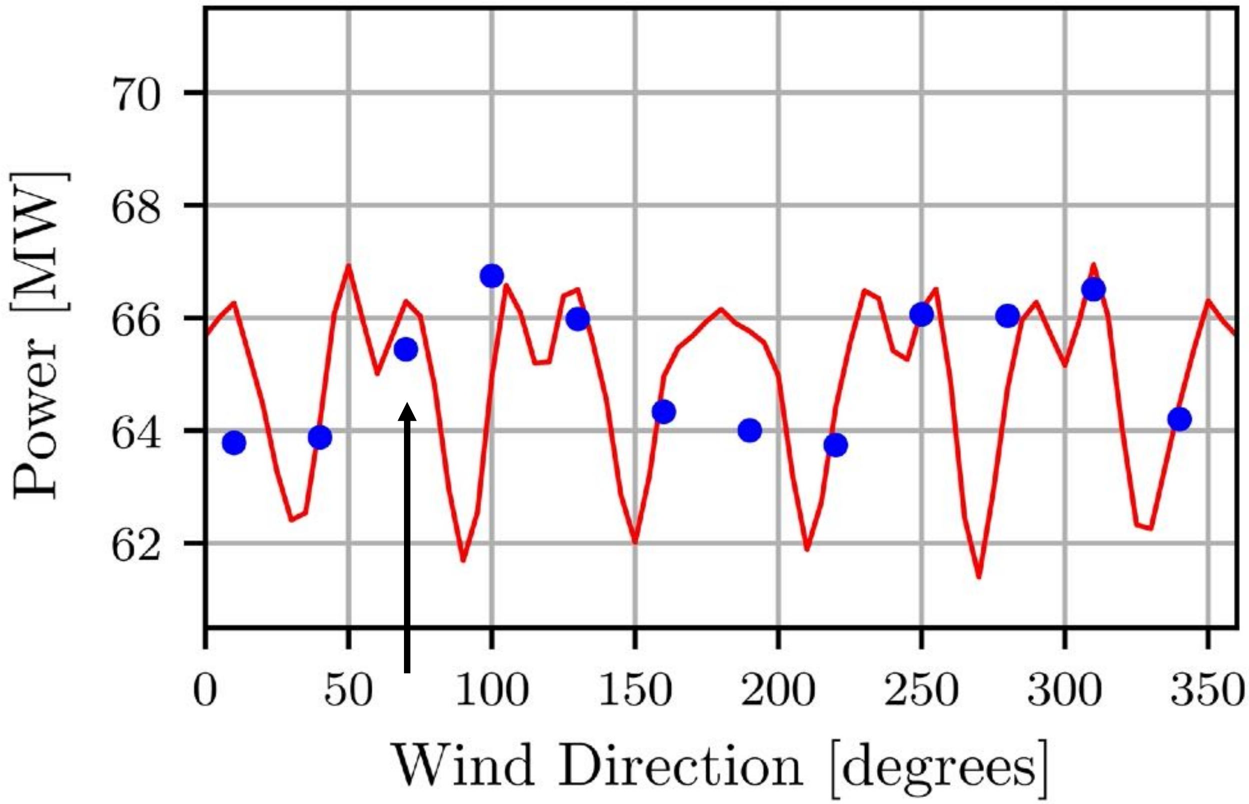






Total wind farm power:

Comparison with data with SOWFA LES (blue) and ALC model (red)

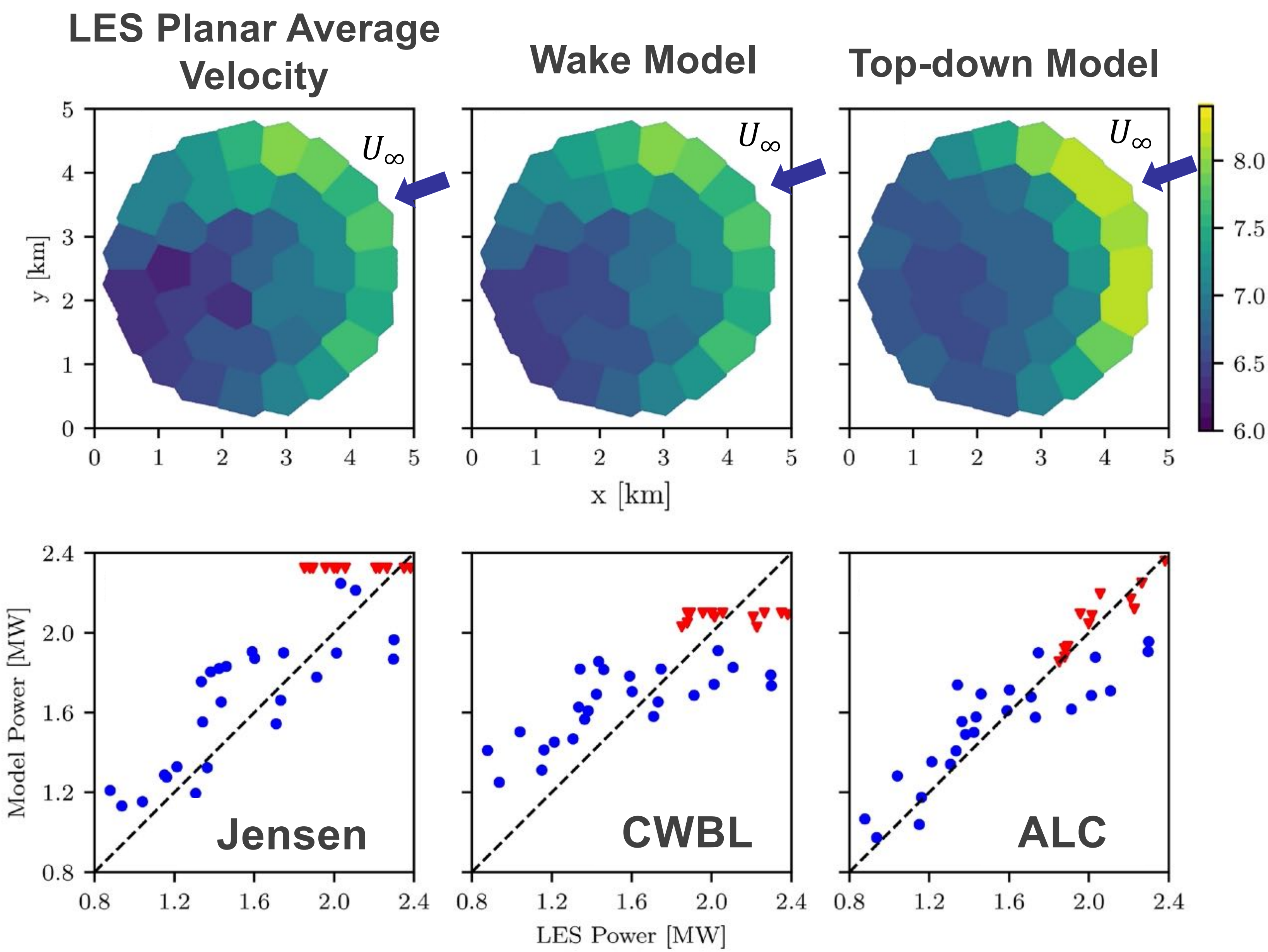


← **1.6 W/m²**

Deeper dive for 70 degrees….

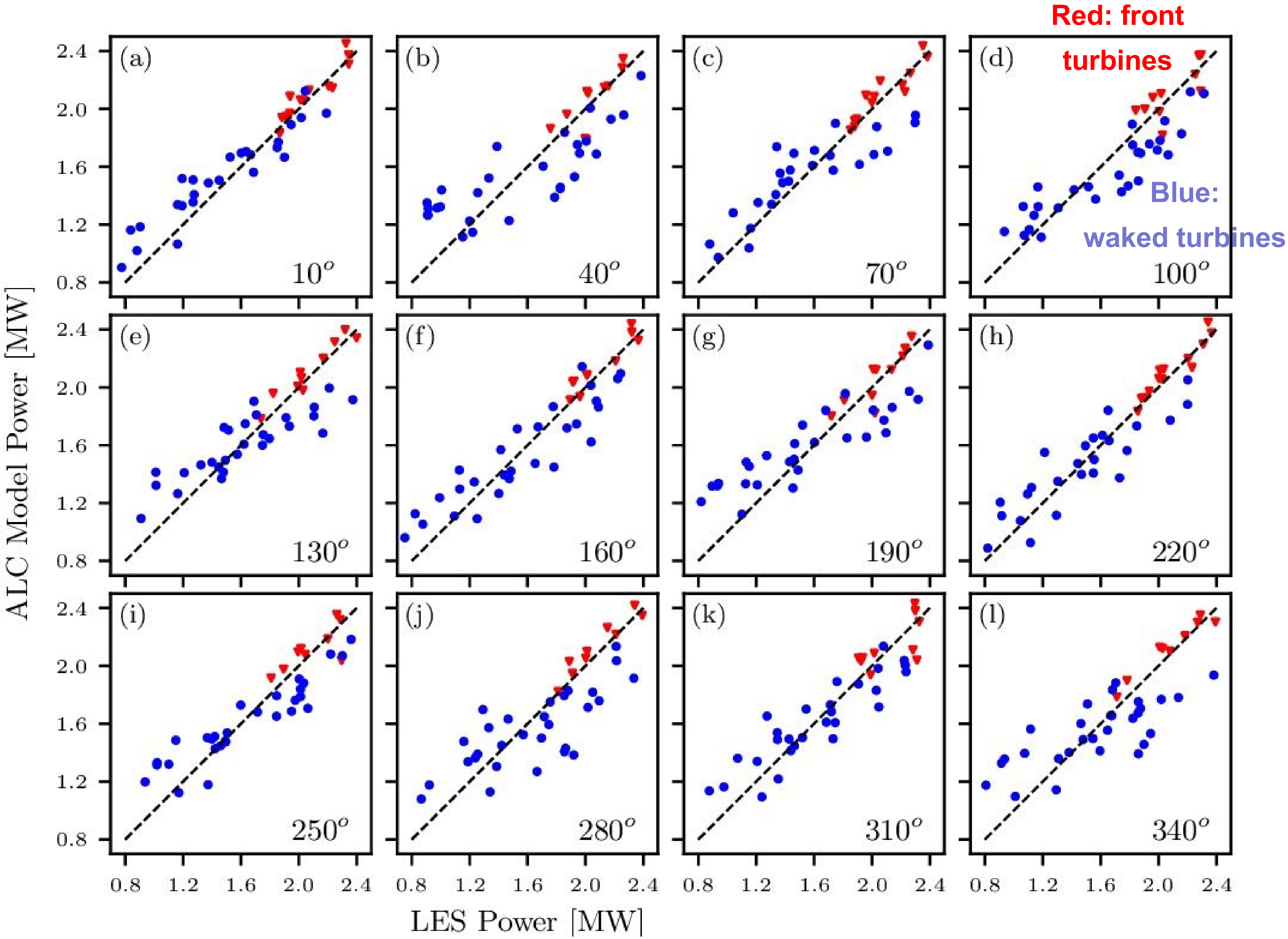
Average distance = $(\pi\ 2000^2/40)^{1/2}$ = 560 m = 4.5 D (low!)

Comparison for 70 degrees



LES not fully converged

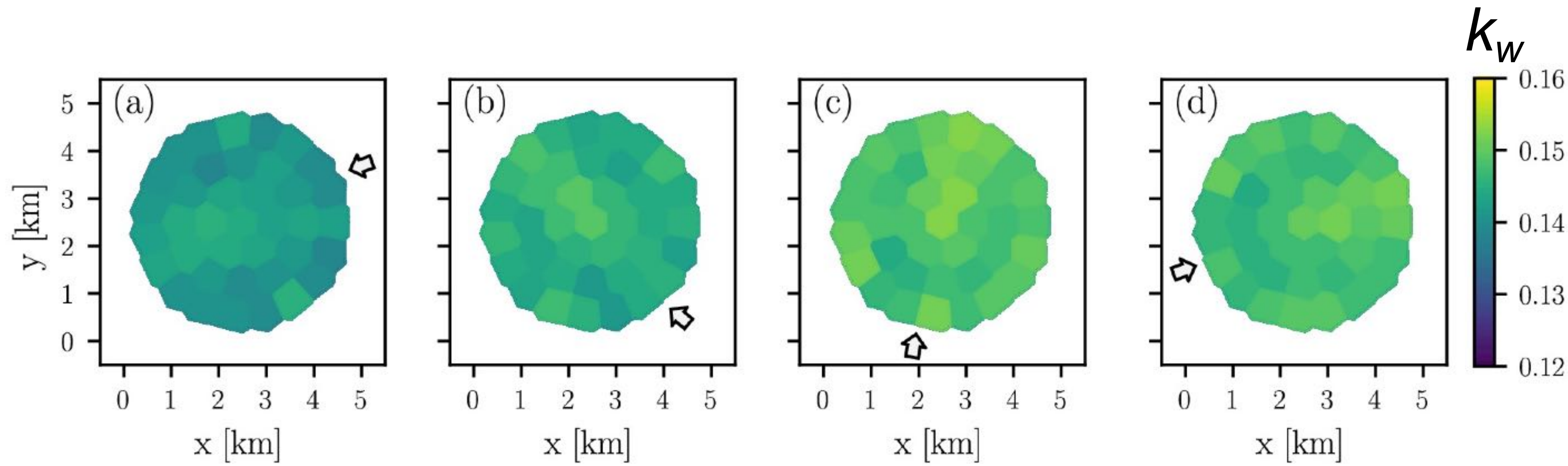
Turbine power comparison for all angles



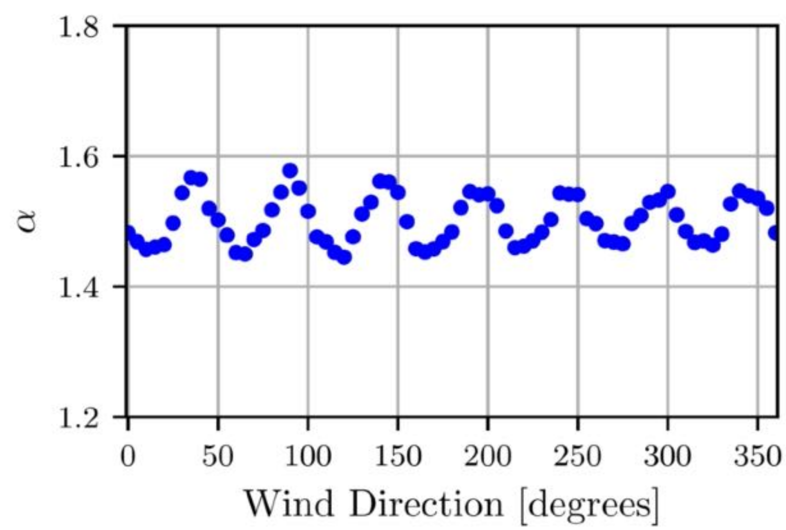
LES not fully converged

Wake expansion coefficients:

$$k_w = \alpha \frac{\frac{1}{2}[u_{*hi}(x) + u_{*lo}(x)]}{\overline{u}_h^{td}}$$

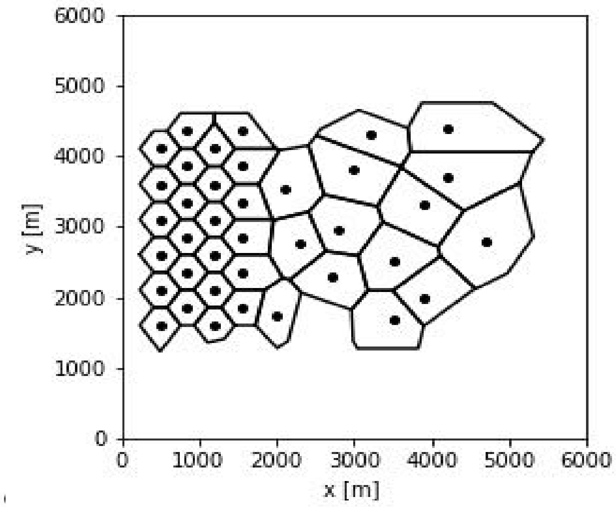


The optimized parameter α for all angles

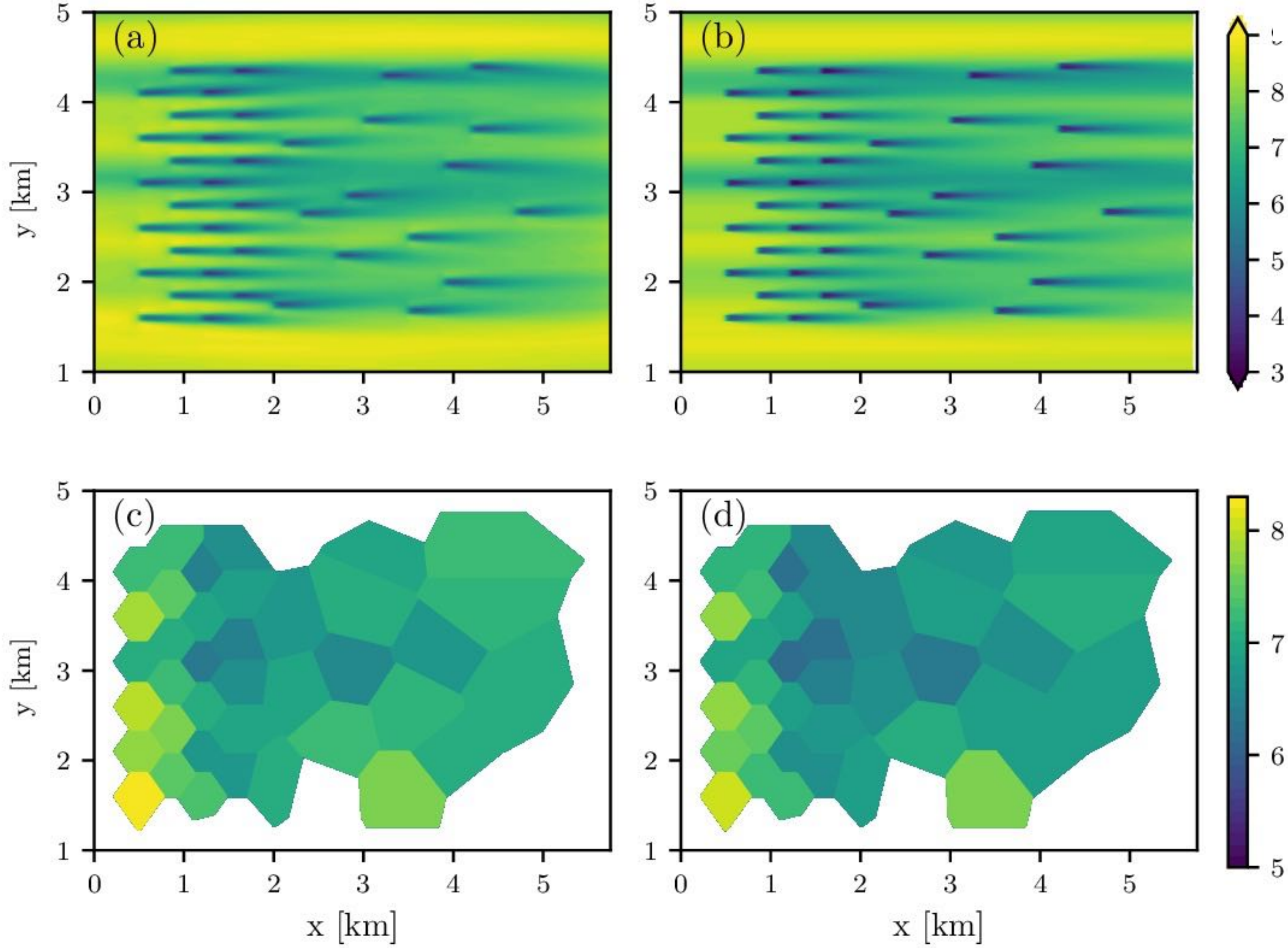


Hybrid regular/random wind farm

- Staggered at the front and 14 random turbines at the rear of the farm
- Run using the JHU LESGO code with an actuator disk model, avrg for ~10 flow-through times



LES **ALC Model**



Hybrid regular/random wind farm

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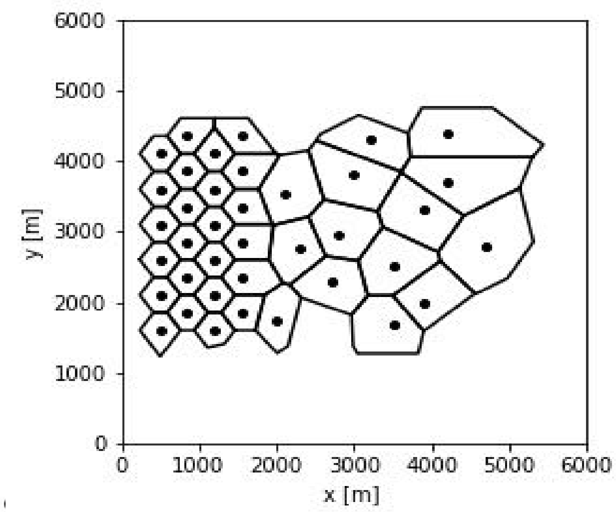
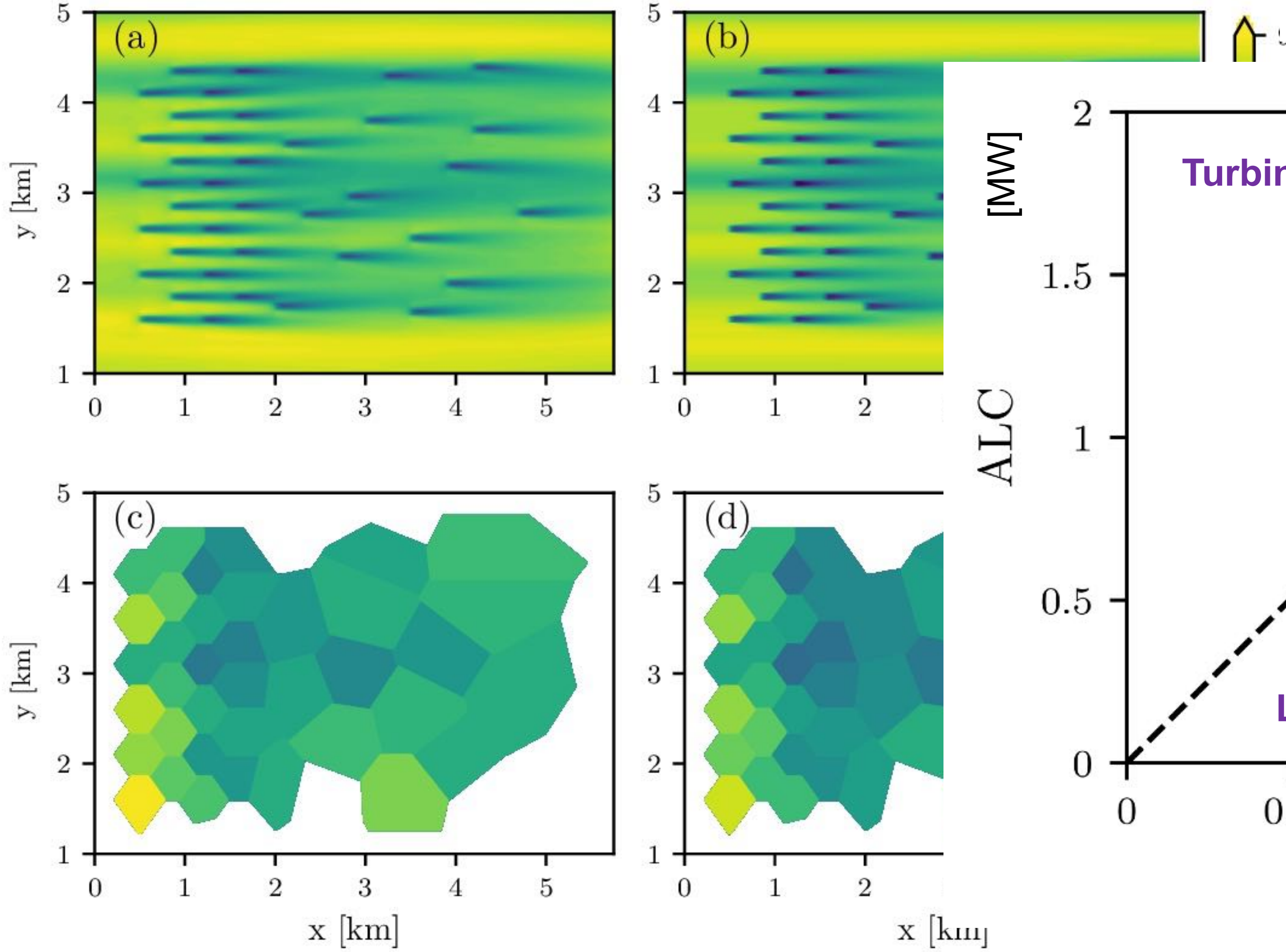
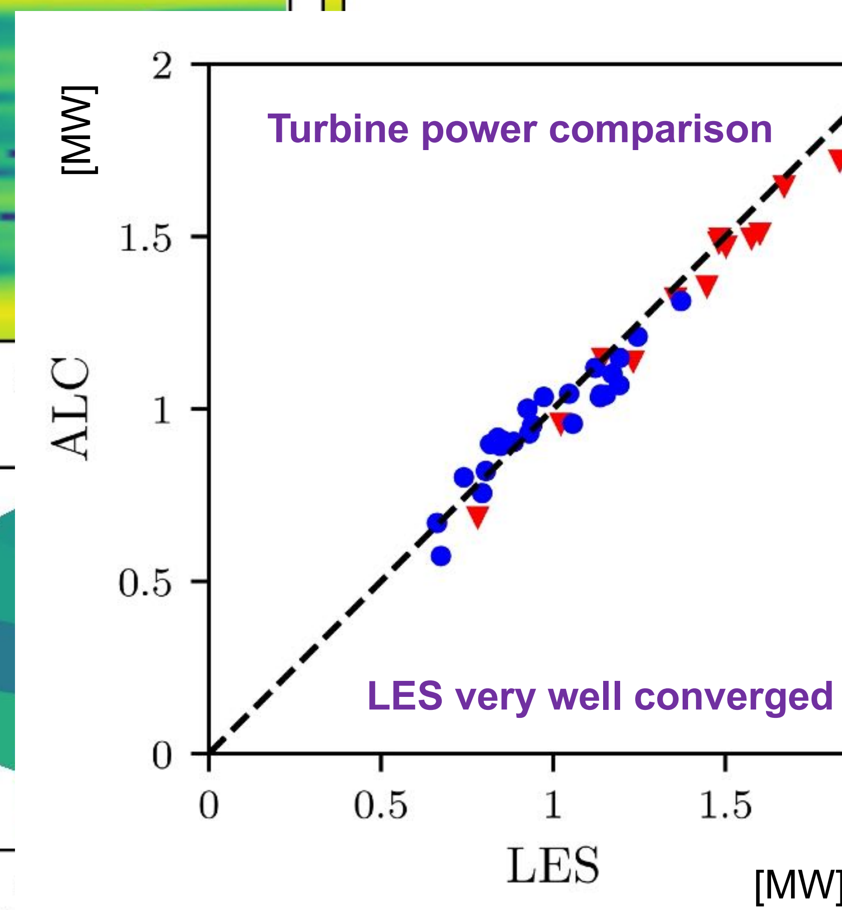


LES

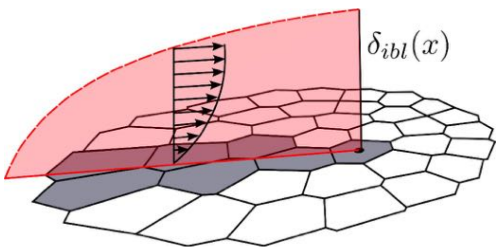
ALC Model

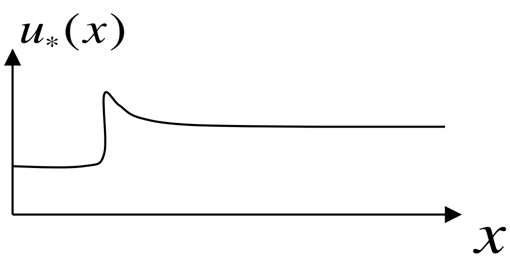


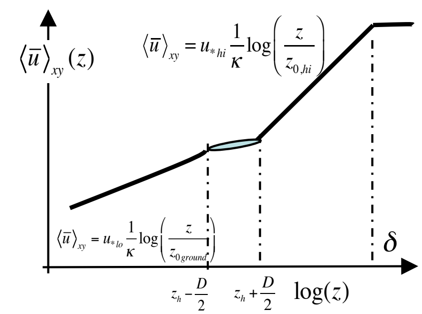


Summary and conclusions:

- Larger and larger wind farms (also off-shore, floating for Pacific West Coast of Americas) being considered
- Fluid dynamics and turbulence provides crucial insights and models, such as :

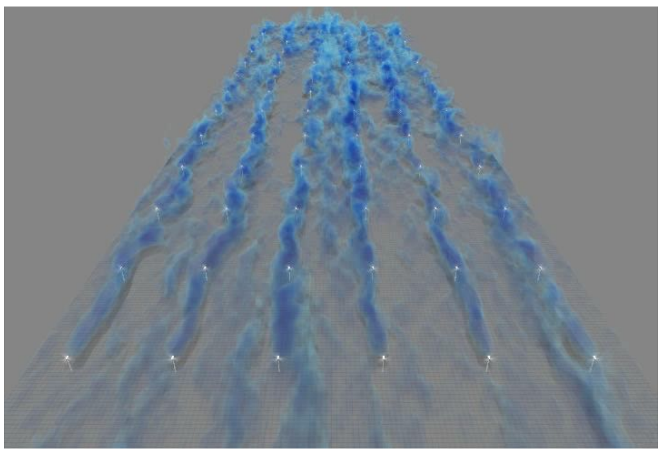


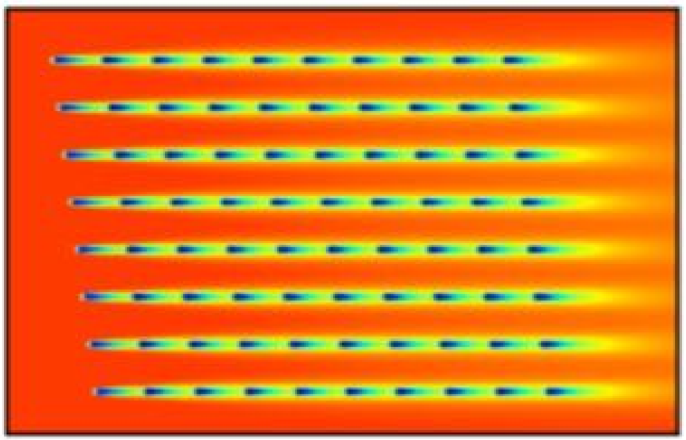




- Double logarithmic velocity profiles in developing IBL
- Coupling of both descriptions: Wake growth rate
- Voronoi tessellation: ALC can predict any layout
- Generalized classical boundary layer concepts ⇨ engineering models

Thank you





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